(1)

A SUMMER PROGRAM IN MATHEMATICS

AND

COMPUTER SCIENCE

FOR

ACADEMICALLY ORIENTED STUDENTS

JUNE 25 - JULY 27, 1990

FUNDED BY
THE OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY

AT

UNIVERSITY OF THE DISTRICT OF COLUMBIA
WASHINGTON, D. C.

Final Report
of
A Summer Program in Mathematics and Computer science

for

Academically Oriented Students
June 25 - July 27, 1990

Funded by
The Office of Naval Research
Department of the Navy

Report by
Bernis Barnes

At
University of the District of Columbia
Washington, D. C.

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Introduction

The 1990 summer program in mathematics and computer science for academically oriented students provided a five-week intensive and rigorous intervention program for thirty-nine (39) students from the D.C. area who are passing to the ninth or tenth grade. As in the previous eight years of the program, addressing the problem of the under-representation of minorities, especially Blacks and Hispanics in engineering, natural sciences, and other mathematics-based fields, was given a high priority. During the summer, the students most of whom are from the under-represented groups were encouraged to strengthen their background in mathematics and to pursue careers in mathematics-based fields. They were also exposed to career opportunities in mathematics-based fields, and to how they should prepare themselves in high school to increase their career options by the time they reach college. The program faculty which has years of experience in teaching local minority students provided the encouragement and motivation, as well as, a carrying and supportive environment.

Purpose and Goals

The purpose of this project is to implement an intensive pre-college intervention program for academically talented students, mainly ninth and tenth grade students from the District of Columbia area, that is designed to increase their representation in mathematics-based careers. By offering the program at this grade level, the students are able to take more rigorous math courses while in high school.

The goals of the project are (1) to strengthen the students' backgrounds in mathematics, computer science, and statistics and operations research, (2) to improve their academic skills with an emphasis on reasoning competencies, (3) to increase their awareness of careers in mathematics-based fields and of the preparation needed for these fields, and (4) to encourage and motivate these students to enroll in calculus track courses while in high school.

The five basic competencies in the area of reasoning are (1) the ability to identify and formulate problems, as well as, the ability to propose and evaluate ways to solve them; (2) the ability to recognize and use inductive and deductive reasoning, and to recognize fallacies in arguments; (3) the ability to draw reasonable conclusions from information found in various sources whether written, spoken, tabular, or graphic, and to defend one's conclusions rationally; (4) the ability to comprehend, develop and use concepts and generalizations; (5) the ability to distinguish between fact and opinion.

Program Personnel

The program staff consisted of long-time members of the faculty of the University. They therefore have had considerable experience in working with local students -- encouraging and motivating them, as well as, providing the needed carrying and supportive environment. Members of the staff were Professors Bernis Barnes and Vernice Steadman, of the Mathematics Department, who taught the General Mathematics classes and coordinated the career education component; Professor William Rice, of the Mathematics Department, who taught the Statistics and Operations Research classes; Professor Gail Finley, of the Computer Science Department, who taught the Computer Science classes and served as assistant director.

Participants

The Participants were selected from a pool of applicants most of whom were recommended by their current mathematics teacher. Five students were recommended by the director of the pre-engineering program of the D.C. public schools. The students were rated on sixteen items of a questionnaire which addressed attitude, achievement, interest, abstract reasoning, study skills, etc. (The complete set of items appear in Appendix B.)

Using the information provided, thirty-nine students were selected -- twenty-one females and eighteen males. Nine of the students were from private or parochial schools and thirty were from public schools; twenty-two had recently completed the eighth grade, and seventeen had recently completed the ninth grade; thirty-five participants were black, one was white, two were Hispanic, and one Asian. The ages of all but six of the students were 13 and 14. Of the remaining six students, two were 12, three were 15 and one was 16.

Program Components

There were two major program components: (1) the academic component which consisted of three courses -- General Mathematics, Computer Science, and Statistics and Operations Research -- which were designed to improve the students' backgrounds in these fields while enhancing their reasoning skills; and (2) the career education component which was designed to provide participants an opportunity to experience "mathematics at work" in every day life through films, videos, etc., and to visit the workplace of and to interact directly with professionals in mathematics-based fields through scheduled field trips and a forum.

Academic Component

The academic component consisted of the three courses. Both the computer science and statistics courses met for one hour and fifteen minutes each morning, and the mathematics course met for one and one-half hours each afternoon, except on Fridays. The computer classes were scheduled for the computer laboratory (a classroom with eighteen IBM PC's with graphic capability and several terminals) three days a week, and the statistics classes were scheduled for the laboratory two days a week. However, the computer classes met in the laboratory most of the time.

The goals, topics and some exercises of each of the courses follow. (Sample curriculum materials used in the courses appear in Appendix C.)

General Mathematics

This course used finite mathematical systems to introduce advanced topics in mathematics. For example, in abstract algebra, the student constructed operation tables for the transformations ("rigid motions") of 3 x 3 magic squares and of equilateral triangles. They also determined the effect of each group property on the rows and columns of finite operation tables. In topology, the students counted the number of topologies on a given finite set. They also identified cluster and interior points, as well as, the closure and boundary of a set for a given finite set.

Also, each student was required to complete a calendar of twenty-five problems, one problem for each day in the program. The solutions to the problems required knowledge of concepts and procedures used in general mathematics, algebra and geometry.

In addition to increasing the students' knowledge, understanding and skills of the topics offered, the goals included the improvement of their

- skills in recognizing patterns and drawing conclusions,
- facility with the technical language of mathematics,
- knowledge of the structural nature of mathematics, and
- skills in techniques of formulating and solving problems.

Most of the topics taught in the course were not topics that are usually taught to students at this level. But, in general, the students performed well and showed interest in the subject matter.
Course Content:

Recognizing Patterns
Deductive and Inductive Reasoning
Functions Defined by sets of Ordered Pairs and by Rules
Properties of Groupoids and Groups on Finite Sets
Cyclic Groups and Generators, Cosets, Normal Groups, and Factor Groups
Endomorphisms on Finite Groups, and the Operation Tables of Endomorphisms
Topologies on Finite Sets, the Cluster Points, Interior Points, Boundary Points, and Closure of the Subsets
Continuous Functions on Finite Sets

Some of the classroom and homework exercises used in the mathematics course are as follows.

. Given worksheets with a variety of patterns and logic exercises, the students identified the patterns and derived logical conclusions. Also, the students were required to recognize patterns and derive logical conclusions throughout the course.

. Given two finite sets A and B, the students identified subsets of the Cartesian product which were
    (a) functions of A into B,

(b) functions of A onto B,
(c) functions of A into B which were 1 to 1,
(d) relations that were not functions, and
(e) constant functions.

. Given collections of ordered pairs that defined functions, the students
(a) identified the domain of the function,
(b) identified the image of the function,
(c) gave the inverse image of the function, and
(d) stated whether or not the inverse of the function was itself a function and justified their answers.

. Given a table of three integral values of a linear function, the students
(a) wrote corresponding equation of the function,
(b) gave the slope of the line,
(c) sketched the graph of the function, and
(d) gave the x and y intercepts of the line.

. Given several graphs of relations, the students identified those graphs which represented functions.

. Given information regarding the slope of a line, such as positive slope, negative slope, slope of '0', slope does not exist, the students gave the direction of the corresponding line.

. Given a function $f:X \to Y$ where X and Y are finite sets, the students listed the elements of $f[X]$ and of its inverse image $f^{-1}[Y]$, and showed that $f[A \cup B] = f[A] \cup f[B]$ and that $f^{-1}[A \cup B] = f^{-1}[A] \cup f^{-1}[B]$.

. Given a 3 by 3 array of squares, the students listed all the 3 by 3 magic squares, established relationships between the magic squares, and constructed an operation table for the composition of the transformations of the magic squares.

. Given the definition of Roup (a group without the associative property), the students identified examples and non-examples in a set of exercises, and justified their answers.

. Given a set with three elements, the students determined the number of groupoids which can be defined on the set, and constructed tables of operations which satisfied a given property or given properties.

. Given the operation table for the composition of the symmetries of the square, the students determined subgroups, cyclic subgroups and their generators, left and right cosets, normal subgroups, and factor groups.

. Given a finite group with three or four elements, the students determined the number of group endomorphisms. Given two finite groups, the students determined which were homomorphic; and when they were, they determined the homomorphism.

- Given the subsets of a three element set, the students identified each collection of subsets of the set which defined a topology on the set.

- Given a topology on a three element set, the students determined the limit points, interior points, boundary points and the closure of each subset of the set.

- Given a function f:X->X on a finite set X and a topology G on X, the students determined the points at which the function was continuous.

Method of Teaching

A variety of teaching strategies were used in the mathematics course. Although most of the sessions were student centered and discovery oriented, lecture demonstrations were given when appropriate. Worksheets were used to focus and guide the classroom sessions, and to provide homework assignments.

Computer Science

The computer course used the fundamentals of programming, flow charting, and computer languages (BASIC, Pascal) to introduce the computer as a tool to aid in problem solving. Throughout the summer, each student had access to personal computers and computer terminals in a laboratory type hands-on experience. The equipment was used to test and run the programs the students wrote, and to run some computer packages.

The goals of the computer course were to prepare students

- to be literate in the language and hardware of computer science,
- to develop algorithms using flowcharts and pseudo code,
- to understand more complex algorithm development using top-down structured design, and
- to construct and debug programs in the BASIC and Pascal languages that employ control statements, string variables and arrays and that use data files, graphic techniques and subroutines.

Course Content:

Unit I: Introduction/Hardware Components
- Orientation and tour of Campus Computing Facilities
- Terminology, History
- Introduction to BASIC
- Lab Session: PC and Terminal Operation, Run Sample Programs

Unit II: Simple Algorithm Development and Implementation
- Flowcharts vs. Top-down Design
- Simple BASIC Programs (INPUT; LET; PRINT; READ..DATA)
- Numeric and Character String Variables
- Lab Session: Simple Programs with INPUT, Arithmetic Computation and OUTPUT; Programs with String Variables

Unit III: Decision and Control Structures
- Use of Pseudo Boolean Variable
- IF..THEN; IF...THEN...ELSE
- Looping and Iteration; FOR...NEXT; WHILE..NEXT Control by out of Data; Count Control; Flag or Marker in Data
- Accumulation
- Lab Session: Programs with Decisions, String Variables; Programs Requiring counting and Tallying

Unit IV: Top-down Structured Design and Problem Solving
- Subroutines (Procedures, Modules)
- GOSUB...
- Lab Session: Programs with Skills to Date

Unit V: Arrays and Tables
- Use of Subscripted Variables in Single Arrays
- Using External Data Files
- File Processing
- Lab Session: Assigned Problems with array Implementation Searching and Sorting

Programming Language Tool: The BASIC language is the one used in most schools at the junior to early senior high school level -- the level of the course participants -- and is the most available language on personal computers. In addition, it is an excellent language for a first exposure to computers. BASIC is therefore the primary language tool used for this course; the enhanced implementation on the VAX 8650 computer system or that of personal computers used at the University is desirable.

Algorithm Development: Many BASIC textbooks use the flow chart as the principle method of algorithm presentation. Therefore, students do need to understand flowcharts and need to be able to use them where practical. The BASIC language itself, in its original form, lends itself to coding from algorithms in flowchart form. However, in recent years, top-down structured design for algorithm development has become more widely accepted in educational circles as a tool to produce more readable and more error-free programs.

Generally the Pascal language is used for top-down structured programming in the first year college course and in the comparable advanced placement course for high school seniors. In keeping with this accepted norm, many advanced BASIC compilers have incorporated some form of the most essential control structures from Pascal to implement structured programming concepts.

This course used flowcharts initially but also uses the structured approach in an attempt to develop better algorithmic skills and to bridge a large gap generally found between teaching methods usually employed with BASIC and those to be used in later courses.

Method of Teaching

Lecture and lecture discussion methods reinforced by problem worksheets were used. Lab activities provided practical implementation of the concepts introduced in the lectures.

The course participants had various levels of prior computer experience. To accommodate this situation, the more advanced students may moved at their own pace through the earlier assignments to more complex problems and projects. It is not reasonable to expect a beginning student to do a great deal with arrays or to independently do many problems beyond that level while a more advanced student may begin with emphasis in that area and continue further. Some lectures and discussions continued during the lab period for advanced projects. The notion of top-down design and some control structures were relatively new for most participants, therefore, lecture sessions were the same for all.

The textbook was used primarily as a reference. The examples in it are for various computer systems in use at the University; annotation to the examples show the variations in use for popular microprocessor systems.

TEXT: Mandell, Steven L., Introduction to BASIC Programming, Second Edition, West Publishing Co., 1985.

Statistics and Operations Research

The Statistics and Operations Research course used quantitative techniques as tools in decision making. The course focused on analyzing, interpreting and utilizing data. Specifically, the students used statistics techniques to summarize data and to make inference based on that data. They also learned to construct models for research.

The goals of the course were

- exploring techniques of assigning numbers that represent chances or probabilities of various events of interest,
- exploring techniques of organizing, summarizing and displaying data to reveal its patterns and relationships, and
- integrating the computer throughout the course, i.e. the students learned to transfer the skills developed in their computer course into problem solving tools in statistics.

Course Content:

Unit I : Exploring Probability
- Experimenting with Chance
- Knowing our Chances in Advance -- Theoretical

Probability
- Complementary Events and Odds
- Compound Events
  - Multiplying Probabilities
  - Adding Probabilities

Unit II: Exploring Data: An Introduction to Statistics
- Line Plots
- Stem-and-Leaf Plots
- Median, Mean, Quartiles, and Outliers
- Box Plots
- Scatter Plots
- Lines on Scatter Plots
- Smoothing Plots Over Time

Some of the Classroom and homework exercises used in the course are as follows:

. Line plots are quick, simple ways to organize data. From a line plot it is easy to spot the largest and smallest values, outliers, clusters, and gaps in the data. It is also possible to find the relative position of particular points of interest.

. Stem-and-leaf plots are used as a substitute for the less informative histograms and bar graphs. From a stem-and-leaf plot it is easy to identify the largest and smallest values, outliers, clusters, gaps, the relative position of any important value, and the shape of the distribution.

. The median and the mean are single numbers that summarize the location of the data. Neither alone can tell the whole story about the data, but sometimes we do want a single, concise, summary value. The lower quartile, median, and upper quartile divide the data into four parts with approximately the same number of observations in each part. The interquartile range, the third quartile minus the first quartile, is a measure of how spread out the data are.

. The box plots is a useful technique for focusing on the relative positions of different sets of data and thereby compare them more easily.

. Scatter plots are best way to display data in which two numbers are given for each person or item. When analyzing a scatter plot, look for clusters of points, points that do not follow the general pattern, and positive, negative, or no association.

. The scatter plot is also the basic method for learning about relationships between two variables. This topic treats the cases where the interpretation becomes clearer by adding a straight line to the plot.

. Smoothing is a technique that can be used with time series data where the horizontal axis is marked off in years, days, hours, ages, and so forth. We can use medians to obtain smoothed values, and these smoothed values can remove much of the sawtooth effect often seen in time series data. As a result, a clearer picture of where values are increasing and decreasing emerges.

Method of Teaching

The teaching strategies were student centered. Exercise sheets from the texts were used to focus and guide the class work and class discussions, and to provide homework assignments.

TEXTS: Newman, Claire M.; Obremski, Thomas E., and Scheaffer, Richard L., Exploring Probability, Dale Seymour Publications, 1987

Landwehr, James M., and Watkins, Ann E., Exploring Data, Dale Seymour Publications, 1986.

The Career Education Component

This component provided the students an opportunity to increase their awareness of the many career opportunities that are available in mathematics based fields, and of the preparation that is necessary to pursue these careers. One type of activity was the field trips to the Naval Research Laboratory in Washington, D. C. and the David Taylor Naval Ship Research and Development center in Caderock, Maryland where the students visited the workplace of scientists, and interacted with some of them. At the David Taylor facility the students saw tests being conducted in the mile long tank and in the wave generating circular tank. While at the Naval Research Laboratory, Dr. George Carruthers gave a lecture on exploring outer space, and Dr. Patricia Tatem talked to the students about her research in fire retardent substances.

In addition to the tours, students viewed video tapes. One of the video tapes was a motivational video by the National Urban Coalition which features successful blacks such as Frederick Gregory and Elonor Holmes Norton. Another is "The Challenge of the Unknown", developed through a grant to the American Association for the Advancement of Science, shows the role of mathematics in solving many problems in every day life.

A number of former program students who are now in college expressed an interest in talking to our students about what to expect in college. Although only three came, they had a positive impact on our students. One of the students is in a five-year joint engineering program at a small black college and a large university. Another student was graduating from the Naval Academy with a degree in mathematics. The third is attending a small private college in the North-east. They all were very positive about the impact of the program on them.

The career education component also included a career awareness forum. The students and their parents interacted with the panelists which included two of the former students described above. The program for this activity is in Appendix D.

Program Evaluation

As with any academic program, some students' assessment were favorable while other were not as favorable. The majority of the students, however, evaluated the overall program, the instructional component, the teaching materials and the extra-curricular activities positively. Most of the students (97%) felt that the program increased their understanding of mathematics and computer science, and prepared them to perform better in their high school courses; (88%) felt that it increased their appreciation of mathematics and computer science; (79%) felt that it prepared them to reason more clearly in mathematics; and (82%) felt that it inspired them to pursue the more challenging mathematics courses while in high school. About seventy percent (70%) of the students felt that the program made them aware of their career options. Also, about eighty percent (80%) of the students felt that the subject matter was about right, while a majority of them felt that the class size and length of the program were about right. But most of the students felt that the class periods were too long.

The faculty generally agreed that most of the anticipated outcomes of the program were reached: (a) the student became more proficient in reasoning skills; (b) the students became more aware of the inter-relation of mathematics and other sciences; (c) the students became more aware of how the computer is used as an aid in solving problems in statistics and other disciplines, and (d) the students became more aware of the capabilities of UDC and the Navy and the opportunities available to them.

Throughout the program, the students were encouraged to remain in the "calculus track" while in high school, as they would then have more career options available to them when they reach college.

APPENDIX A

A Summer Program in Mathematics and Computer Science

1990

Sigma

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 to 10:10	Prof. Rice Statistics Room 106/Bldg. 32	Prof. Rice Statistics Lab Room 101/Bldg. 32	Prof. Rice Statistics Room 106/Bldg. 32	Prof. Rice Statistics Lab Room 101/Bldg. 32	Prof. Rice Statistics Room 106/Bldg. 32
10:20 to 11:30	Prof. Finley Computer Science Lab Room 101/Bldg. 32	Prof. Finley Computer Science Room 105/Bldg. 32	Prof. Finley Computer Science Lab Room 101/Bldg. 32	Prof. Finley Computer Science Room 105/Bldg. 32	Prof. Finley Computer Science Lab Room 101/Bldg. 32
11:30 to 12:30	Lunch	Lunch	Lunch	Lunch	Lunch
12:30 to 2:00	Profs. Steadman/Barnes Room 105/Bldg. 32 General Mathematics	Profs. Steadman/Barnes Room 105/Bldg. 32 General Mathematics	Profs. Steadman/Barnes Room 105/Bldg. 32 General Mathematics	Profs. Steadman/Barnes Room 105/Bldg. 32 General Mathematics	Feild Trips/Forums July 11, 1990 David Taylor Research Center July 21, 1990 Navy Research Laboratory

A Summer Program in Mathematics and Computer Science

1990

Theta

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 to 10:10	Prof. Finley Computer Science Lab Room 101/Bldg. 32	Prof. Finley Computer Science Room 105/Bldg. 32	Prof. Finley Computer Science Lab Room 101/Bldg. 32	Prof. Finley Computer Science Room 105/Bldg. 32	Prof. Finley Computer Science Lab Room 101/Bldg. 32
10:20 to 11:30	Prof. Rice Statistics Room 106/Bldg. 32	Prof. Rice Statistics Lab Room 101/Bldg. 32	Prof. Rice Statistics Room 106/Bldg. 32	Prof. Rice Statistics Lab Room 101/Bldg. 32	Prof. Rice Statistics Room 106/Bldg. 32
11:30 to 12:30	Lunch	Lunch	Lunch	Lunch	Lunch
12:30 to 2:00	Profs. Steadman/Barnes Room 106/Bldg. 32 General Mathematics	Profs. Steadman/Barnes Room 106/Bldg. 32 General Mathematics	Profs. Steadman/Barnes Room 106/Bldg. 32 General Mathematics	Profs. Steadman/Barnes Room 106/Bldg. 32 General Mathematics	Feild Trips/Forums July 11, 1990 David Taylor Research Center July 21, 1990 Navy Research Laboratory

APPENDIX B

University of the District of Columbia
College of Physical Science Engineering and Technology

Department of Mathematics, MB4203
4200 Connecticut Avenue, N.W.
Washington, D.C. 20008

Telephone (202) 282-3171

April 2, 1990

Dear Principal:

The Department of Mathematics and the Department of Electrical Engineering at the University of the District of Columbia are pleased to announce a program, entitled, **A Five-Week Summer Program in Mathematics and Computer Science for Academically Oriented Students,** Scheduled June 25 through July 27, 1990. This program will provide a five-week, intensive, exciting and rigorous academic program in mathematics, computer science and operations research for forty (40) ninth and tenth grade students and is funded by the office of Naval Research, Department of the Navy.

Please encourage the teachers of mathematics at your school to recommend no more than two students who are capable of success in this highly structured academic program. These recommendations should be postmarked by May 11, 1990. Application forms are included and must be completed by the appropriate mathematics teachers.

Thank you in advance for your prompt consideration. We look forward to hearing from the teachers at your school.

Sincerely,

Bernis Barnes
Project Director

Selection of Participants for the Summer Program

To be considered for this program the student must be:

1. recommended by his/her mathematics teacher.
2. passing to the ninth or tenth grade.
3. motivated to work hard.
4. a serious student.

Stipends

Each student will receive a stipend of $250.00 for participating in the five week program.

Applications

Address applications to:

A Summer Program
Department of Mathematics
University of the District of Columbia
4200 Connecticut Avenue, N.W.
Washington, D.C. 20008

Deadline

Applications must be postmarked no later than May 11, 1990. Students will be notified by May 25, 1990

Further Information

Dr. Beverly J. Anderson 282-3171
Professor Bernis Barnes 282-3171
Dr. Alvin Darby 282-7427

UNIVERSITY OF THE DISTRICT OF COLUMBIA 1976

The Department of Mathematics and The Department of Electrical Engineering

present

A Summer Program in Mathematics and Computer Science for Academically Oriented Students

June 25 – July 27, 1990

Funded by the Office of Naval Research
Department of Navy

A program in mathematics and computer science for academically talented students will be offered at the University of the District of Columbia (Van Ness Campus) this summer. This program will focus on reasoning competencies while enriching the educational experience of the students. Specifically, the program will provide a five-week, intensive, exciting and rigorous academic program in mathematics, computer science, statistics and operations research for forty (40) ninth and tenth grade students.

The Department of Mathematics and the Department of Computer Science realize that many students are capable of success in mathematics-based fields, but they have not been motivated to seek careers in those areas. We recognize the need to provide exciting programs in mathematics and computer science to intrigue these students and stimulate their interest in mathematics and mathematics-based fields. This need is most acute where the students may suffer from educational, financial, or cultural disadvantage.

This program is open to all students, without discrimination.

CURRICULUM SPECIFICS

General Mathematics

Finite mathematical systems will be used to introduce structure in algebra, topology and geometry. The student-centered classes will be designed to encourage students to investigate these topics. The focus will be on the language, patterns and logical nature of mathematics

Computer Science

The fundamentals of programming, flow charting and the BASIC language will be used to introduce the computer as a tool to aid in solving problems in many disciplines. Each student will have access to a computer terminal in a laboratory type hands-on experience.

Statistics and Operations Research

Students will use quantitative techniques as a tool in decision making. They will study statistical and OR techniques and apply them to management type problem solving. This component will be computer based and will focus on interpretation and utilization of data.

Field Trips, Films and Forums

Field .rips, films or forums will be scheduled on Fridays. These opportunities will be provided for students to experience the use of mathematics in the working world of the scientist and for students to interact with professionals in the field. The Office of Naval Research will work cooperatively with UDC in implementing this component.

Application Form

SUMMER PROGRAM IN MATHEMATICS AND COMPTUER SCIENCE

(To be completed by Mathematics Teacher)

Please Print or Type

____________________________________ ( )Male ( )Female ________________

Student's Last Name First Middle Social Security No.

Present Grade 8th ( ) 9th ( ) Date of Birth________________

____ __________________

Parent/Guardian's Last Name First Middle Home Telephone

Home Address____

Number & Street City State Zip Code

____

Title of Mathematics Course in which student is currently enrolled

1) What is the best estimate you can give to the applicant's present rank in your course?

   Top 10% 2nd 10% 3rd 10% 4th 10% 5th 10%

2) What is the applicant's attitude toward and interest in the course work?

   Outstanding Excellent Good Average Below Average

3) What are the levels of promptness and attention to detail with which class assignments are completed by the applicant?

   Outstanding Excellent Good Average Below Average

4) What is the applicant's level of abstract reasoning?

   Outstanding Excellent Good Average Below Average

5) What is the applicant's level of computational skills?

   Outstanding Excellent Good Average Below Average

Please check the appropriate section.

	Outstanding	Excellent	Good	Average	Below Average
Study Habits					
Self Motivation					
Organization of Time & Work					
Intellectual Curiosity					
Attention Span					
Ability to express ideas orally					
Ability to follow directions					
Ability to work independently					
Perseverance					
Attendance					
Parent Cooperation					

(Over)

Comments:

Teacher's Name (Print or Type)

Teacher's Signature

School

School Phone Number

Please Return This Form
by May 11, 1990

A Summer Program
Department of Mathematics
University of the District of Columbia
4200 Connecticut Avenue, N.W.
Washington, D. C. 20008

NAVY PROGRAM
1990
Final Roster

Sigma

1. Nadir Al-Salam
2. Gimbu Bandele
3. Shani R. Boone
4. LaKita A. Crider
5. Rachael Franklin
6. Lance E. Gurley
7. Miasia Johnson
8. Franesha Latimer
9. Angel Maldonado
10. Nikita Powell
11. Charlita Rodney
12. Mary Rose
13. Eric Royster
14. Niesha Shelto
15. Jose Sorto
16. Char Stoddard
17. Pynette Thomas
18. Dionte Y. Williams
19. Tamara Williams

Theta

1. Sean Ambush
2. Jacqueline Bates
3. Tamie Blow
4. Russell M. Davis
5. Sarah Elwell
6. Taray Green
7. Adrian W. Harris
8. Jonus Johnson
9. Kenneth Keels, Jr.
10. Ebony Matthews
11. Jamie Morse
12. Gail Potter
13. Raymond Prince
14. Rafel R. Rosier
15. Torrise L. Ruffin
16. Diantre S. Smith
17. Lachelle Spence
18. Julian Sutton
19. Kenneth Wijesinghe
20. Michelle Williams

Staff

Bernis Barnes
Vernise Steadman
Darrell Chatmon
Gail Finley
William Rice
Leonard Dunning

A Summer Program in Mathematics and Computer Science
Orientation Session
June 25, 1990

Welcome	Professor Bernis Barnes Program Director
Greetings	Dr. Philip L. Brach Dean - College of Physical Science Engineering & Technology
Introduction of Faculty, Staff and Students	
Overview of Academic Program	Professor Bernis Barnes Professor Gail Finley Professor William Rice
Expectations of the Program	Professor Bernis Barnes
Tour of Facilities at UDC	Professor William Rice Professor Gail Finley
Photo Identification Session	Mr. James Stephens Building 38, 2nd Floor Lounge

Follow your afternoon schedule - 12:30 until 2:00

A Summer Program in Mathematics and Computer Science

June 26 - July 28, 1989

Purpose

To provide a five-week, intensive and rigorous academic program in mathematics (including probability and statistics), computer science and operations research for forty (40) academically-talented ninth and tenth grade students drawn from the District of Columbia.

This program will focus on applications of reasoning competencies as delineated by the College Board. The program will concentrate on enriching the educational experiences of the students and hopefully will motivate them to pursue the more rigorous mathematics courses in their remaining high school years.

The University of the District of Columbia and the Office of Naval Research of the Department of the Navy are concerned about increasing minority representation in mathematics and mathematics based fields.

Personnel

Professors:

Bernis Barnes
282-3171

Gail Finley
282-7345

William Rice
282-3171

Vernice Steadman
282-3171

Administrative Assistants:

Wilma Thompson
282-3171

Yingying Zhou
282-3171

Assignments

Students are expected to complete all assignments. Homework assignments are carefully selected to reinforce concepts presented in class.

Attendance

Students are expected to meet their schedules daily and on time. Tardiness and absenteeism will surely interrupt the continuity of the subject matter so carefully prepared for the students.

Excused absences may be obtained from the Department of Mathematics at 282-3171. Parents or guardians should call the office between 8:30 and 9:00 on the day of absence.

Field Trips

The instructional phase of the program will be supplemented by field trips, films and a forum scheduled on Friday afternoons. The field trips and films will allow students an opportunity to see the use of mathematics in the working world of the scientist, and the forum will present a situation for students to interact with professionals from the Navy and the University of the District of Columbia. Parental consent is required for students to attend the field trips.

Attire

Students are expected to wear modest attire.

No Nos

1. No radios
2. No gum
3. No tardiness
4. No food or drink in the classrooms or computer laboratories
5. No smoking

Student Identification Cards

All students will be issued a student identification card on June 26, 1989. the ID card is required for use of all university services and must be available for presentation to security personnel in university buildings. **Please wear your student ID card throughout the course of the program.**

Health Appraisal Form

The University Health Center is authorized to provide services to minors with parental consent. Parents desiring the service should so indicate on the Health Appraisal Form. The University Health Service is located in Building 44 Room A33 and directed by Dr. Franklin.

Services for Students

Bookstore - The university bookstore is located in Building 38, Level A. The bookstore is opened from 9:00 a.m. until 5:00 p.m. and provides books and supplies that might be needed by the students. Snacks are also available in the bookstore.

Library - The university maintains four libraries. The main collection is located on the Van Ness Campus in Building 41, Level A. The collection includes more than 400,000 books and more than 1,000,000 items including microfilms, media materials and government documents. There are reading rooms, open stacks, microfilms and individual study carrels.

The hours of the Van Ness Library are between 8:00 a.m. and 7:00 p.m. A valid university ID is required by students using the services of the library.

Computer Facilities - Computer facilities are available to students in Building 32, Room 101 and in Building 41, room 302. Students will have access to the university's computer systems via CRT and printer terminals.

Eating Facilities - Students may bring bag lunches daily. Bag lunches should be brought on days of field trips, as the trips are scheduled shortly after the morning classes.

Eating facilities in the area include the university cafeteria, located in Building 38, Level B and several fast food establishments within two blocks of the university, i.e., Roy Rogers, Burger King, and Vie De France -- all are located on Connecticut Avenue.

Permission Slip to Take Field Trips

My child is a student in the Navy sponsored program entitled, **A Summer Program in Mathematics and Computer Science.** I am aware that two field trips have been planned for the students in the program at no cost to them.

I am also aware that bus transportation will be provided by the University of the District of Columbia for the field trips.

My child ______________________________ has permission
Name of Child

to participate in the following field trips.

1) **David Taylor Research Center**
Caderock, Maryland 20084
July 11, 1990
Noon - 4:00 P.M.
Please bring bag lunch

____
Signature of Parent or Guardian Date

2) **Naval Research Laboratory**
4555 Overlook Avenue, S.W.
Washington, D.C.
July 21, 1990
Noon - 4:00 P.M.
Please bring bag lunch

____
Signature of Parent or Guardian Date

Please return by June 29, 1990

IMMUNIZATION RECORD OF ____

Last Name First Middle

DATE OF BIRTH ____

IMMUNIZATION VACCINE	Date Given Mo/Da/Yr	Specify Type	Validate below by Physician/Clinic	Date Dose Due
POLIO	1			
	2			
	3			
	4			
	5			
DPT/DT Td	1			
	2			
	3			
	4			
	5			
MEASLES				
RUBELLA				
MUMPS				
OTHER				

TB TESTS

MEDICAL EXEMPTION: Reason: ____

Physician's Signature ______________________________ Date __________

Comments/Notes:

APPENDIX C

General Mathematics

UNIT: TOPOLOGICAL STRUCTURES ON SETS

This unit focuses on the topological structures on sets and on the technical language that is used in these systems. The topics are functions, point set topologies, basic topological concepts, and continuous functions on finite sets.

The objectives for the students are

To identify and establish relations that are functions and functions that are onto and/or one-to-one, and to determine the image and inverse image of given sets for the given functions.

To determine if a given collection of subsets of a set is a topology on that set, and to form collections of subsets of a given set that are topologies on that set.

To identify the interior, exterior, boundary and cluster points of a given set, and the complement and closure of that set for the given topology.

To use the definition of a continuous function to show that a given function is or is not continuous at a given point for the given topology.

This unit will be taught in four lessons. In teaching each lesson, there will be emphasis on recognizing patterns, using reasoning skills, and interpreting and applying given definitions. The attached worksheets will be used to structure, focus and guide the classwork, and to provide homework assignments.

<u>Technical Terms</u>

set	relation
element	function
subset	into
equal sets	domain
union	range
intersection	codomain
complement	onto
cluster point	one-to-one
interior point	image
exterior point	inverse image
boundary point	topology
closure of a set	open set
one and only one	continuity
if and only if	basis

RELATIONS AND FUNCTIONS ON SETS

1. The Cartesian product of sets A and B is the set of all ordered pairs such that the first element of each pair is an element of A and the second element of each pair is an element of B (written $A \times B$).

   If $A = \{a,b,c\}$ and $B = \{1,2\}$, list the elements of $A \times B$ and the elements of $B \times A$.

   Also, list the elements of $A \times A$.

2. A relation from set A into set B is a subset of $A \times B$.

   If $A = \{a,b,c\}$ and $B = \{1,2\}$, define five relations from set A into set B.

   How many relations are there from set A into set B?

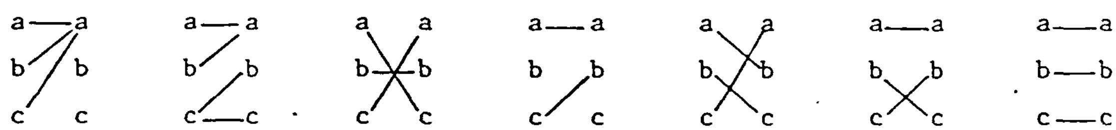
   How many relations are there from set A into set A?

3. A function from set A into set B is a relation from A into B in which each element of A is paired with one and only one element of B.

   i. The set of all first components of a function, all the elements of A, is called the domain of the function.

   ii. The set of all second components of a function, a subset of B, is called the range of the function.

   Which of the following relations define functions on the set $A = \{a,b,c\}$?



   What is the range of each of the functions?

   How many functions are there from set A into set A?

   If $A = \{a,b,c\}$ and $B = \{1,2\}$, how many functions are there from set A into set B?

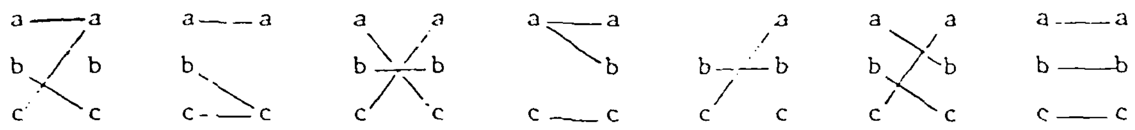
   If set A has n elements and set B has m elements, determine the following:

   a. How many relations are there from set A into set B?

   b. How many functions are there from set A into set B?

Def. The function $f:A \to B$ is ONTO iff for each element $y \in B$, there is an element $x \in A$ such that $f(x) = y$.

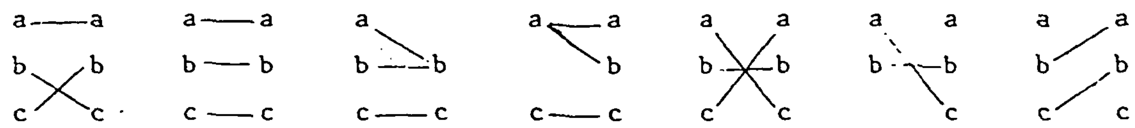
a. Which of the following functions are ONTO functions?



List the remaining ONTO functions $f:V \to V$ where $V = \{a,b,c\}$.

Def. The function $f:A \to B$ is ONE-TO-ONE iff $f(x) = f(y)$ implies that $x = y$.

b. Which of the following functions are ONE-TO-ONE functions?



List the remaining ONE-TO-ONE functions of $f:V \to V$ where $V = \{a,b,c\}$.

. Def. If $f:X \to Y$, then

$f[X] = \{f(x) \in Y : x \in X\}$ is the image of X in Y, and

$f^{-1}[Y] = \{x \in X : f(x) \in Y\}$ is the inverse image of X in Y.

Let $X = \{1,2,3,4,5\}$ and $Y = \{1,2,4\}$, and define $f:X \to Y$ by $f(2) = 1$, $f(4) = 2$ and $f(1) = f(3) = f(5) = 4$. Let $A = \{1,2,3,4\}$ and $B = \{1,2,5\}$.

a . Find:

$f[A] = f[A \cap X] =$

$f[B] = f[B \cap X] =$

$f[A \cup B] =$

$f[A \cap B] =$

Show that:

$f[A \cup B] = f[A] \cup f[B]$

$f[A \cap B] \subseteq f[A] \cap f[B]$

b. Find:

$f^{-1}[A] = f^{-1}[A \cap Y] =$

$f^{-1}[B] = f^{-1}[B \cap Y] =$

$f^{-1}[A \cup B] =$

$f^{-1}[A \cap B] =$

Show that:

$f^{-1}[A \cup B] = f^{-1}[A] \cup f^{-1}[B]$

$f^{-1}[A \cap B] = f^{-1}[A] \cap f^{-1}[B]$

UNIT: TOPOLOGICAL STRUCTURES ON SETS
Topologies on Finite Sets

Name_______________________

Def. A set X is a subset of a set V iff each element of X is an element of V.

Which of the following sets are subsets of the set {a,b,c}?

{a,b}   {a}   {a,d}   { }   {a,b,c}   {d}

List the subsets of {a,b,c} that are not given above.

Def. Any collection of subsets of a finite set V is a topology on V iff
i). the collection contains the empty set and the set V itself, and
ii). the collection is closed under the operations of union and intersection.

1. The collection {∅, {a,b}, {a,b,c}} of subsets of {a,b,c} is a topology on {a,b,c}.

i). The collection contains ∅ and {a,b,c}.

ii).

∪	∅	{a,b}	{a,b,c}
∅	∅	{a,b}	{a,b,c}
{a,b}	{a,b}	{a,b}	{a,b,c}
{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}

∩	∅	{a,b}	{a,b,c}
∅	∅	∅	∅
{a,b}	∅	{a,b}	{a,b}
{a,b,c}	∅	{a,b}	{a,b,c}

2. Show that the collection {∅, {a}, {b,c}, {a,b,c}} of subsets of {a,b,c} is a topology on {a,b,c}.

3. State the reason why each of the following collections of subsets of the set {a,b,c} is not a topology on {a,b,c}.

{∅, {a}, {b}, {a,b}}   {{a}, {a,b}, {a,b,c}}   {∅, {a,b}, {b,c}, {a,b,c}}

4. Are the following collections of subsets of $\{a,b,c\}$ topologies on $\{a,b,c\}$?

$\{\emptyset, \{a,b,c\}\}$

$\{\emptyset, \{a\}, \{a,b,c\}\}$

$\{\emptyset, \{a\}, \{a,b\}, \{a,b,c\}\}$

$\{\emptyset, \{b\}, \{a,c\}, \{a,b,c\}\}$

$\{\emptyset, \{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$

$\{\emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$

$\{\emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$

$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

$\{\emptyset, \{a,b\}, \{a,b,c\}\}$

5. List the topologies on the set $\{a,b,c\}$ that are not given above. How many topologies can be defined on a set with three elements?

Def. Let T be a topology on set V, then a collection of subsets B of V is a basis for T iff
 i). $B \subseteq T$, and
 ii). every $X \in T$ is the union of members of B.

6. To the right of each topology in number 4 above, give a basis for that topology.

TOPOLOGICAL SETS

For each set X ($X \subseteq V$) in the tables below, list the elements of the following sets.

1. Complement of X: $X^c = \{p \in V: p \notin X\}$

2. Interior of X: $Int_G(X) = \{p \in X:$ there exists a set $A \in G$ such that $p \in A$ and $A \subseteq X\}$

3. Exterior of X: $Ext_G(X) = \{p \in X^c: p$ is an interior point of $X^c\}$

4. Boundary of X: $Bd_G(X) = \{p \in V: p \notin Int_G(X)$ and $p \notin Ext_G(X)\}$

5. Cluster points of X:
   $Cp_G(X) = \{p \in V:$ every set $A \in G$ which contains p contains a point of X other than p$\}$

6. Closure of X: $Cl_G(X) = \{p \in V: p \in X$ or $p \in Bd_G(X)\}$

7. A set X is <u>open</u> relative to G iff every point in X is interior to X. Which of the sets X are open sets?

Let $V = \{1,2,3\}$, complete the following table for the given collection G of subsets of V.

$G = \{\emptyset, \{1\}, V\}$

X	X^c	$Int_G(X)$	$Ext_G(X)$	$Bd_G(X)$	$Cp_G(X)$	$Cl_G(X)$
$\emptyset$	V	$\emptyset$	V	$\emptyset$	$\emptyset$	$\emptyset$
$\{1\}$	$\{2,3\}$	$\{1\}$	$\emptyset$	$\{2,3\}$	$\{2,3\}$	V
$\{2\}$	$\{1,3\}$	$\emptyset$	$\{1\}$	$\{2,3\}$	$\{3\}$	$\{2,3\}$
$\{3\}$	$\{1,2\}$	$\emptyset$	$\{1\}$	$\{2,3\}$	$\{2\}$	$\{2,3\}$
$\{1,2\}$	$\{3\}$	$\{1\}$	$\emptyset$	$\{2,3\}$	$\{2,3\}$	V
$\{1,3\}$	$\{2\}$	$\{1\}$	$\emptyset$	$\{2,3\}$	$\{2,3\}$	V
$\{2,3\}$	$\{1\}$	$\emptyset$	$\{1\}$	$\{2,3\}$	$\{2,3\}$	$\{2,3\}$
V	$\emptyset$	V	$\emptyset$	$\emptyset$	$\{2,3\}$	V

$G = \{\emptyset, \{1\}, \{1,2\}, V\}$

X	X^c	$Int_G(X)$	$Ext_G(X)$	$Bd_G(X)$	$Cp_G(X)$	$Cl_G(X)$
$\emptyset$						
$\{1\}$						
$\{2\}$						
$\{3\}$						
$\{1,2\}$						
$\{1,3\}$						
$\{2,3\}$						
V						

or each set X ($X \subseteq V$) in the tables below, list the elements of the following sets.

. Complement of X: $X^c = \{p \in V: p \notin X\}$

. Interior of X: $Int_G(X) = \{p \in X$: there exists a set $A \in G$ such that $p \in A$ and $A \subseteq X\}$

. Exterior of X: $Ext_G(X) = \{p \in X^c$: p is an interior point of $X^c\}$

. Boundary of X: $Bd_G(X) = \{p \in V: p \notin Int_G(X)$ and $p \notin Ext_G(X)\}$

5. Cluster points of X:
   $Cp_G(X) = \{p \in V$: every set $A \in G$ which contains p contains a point of X other than p$\}$

6. Closure of X: $Cl_G(X) = \{p \in V: p \in X$ or $p \in Bd_G(X)\}$

7. A set X is open relative to G iff every point in X is interior to X. Which of the sets X are open sets?

Let $V = \{1,2,3\}$, complete the following table for the given collection G of subsets of V.

$G = \{\emptyset, \{1\}, \{2,3\}, \{1,2,3\}\}$

X	X^c	$Int_G(X)$	$Ext_G(X)$	$Bd_G(X)$	$Cp_G(X)$	$Cl_G(X)$
$\emptyset$						
{1}						
{2}						
{3}						
{1,2}						
{1,3}						
{2,3}						
V						

$G = \{\emptyset, \{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$

X	X^c	$Int_G(X)$	$Ext_G(X)$	$Bd_G(X)$	$Cp_G(X)$	$Cl_G(X)$
$\emptyset$						
{1}						
{2}						
{3}						
{1,2}						
{1,3}						
{2,3}						
V						

UNIT: TOPOLOGICAL STRUCTURES ON SETS
Continuous Functions

Name________________________

Def. Let G and H be topologies on sets V and U respectively, then the function $f:V \to U$ is continuous at a point $p_0 \in V$ relative to the given topologies G and H iff for every set $Y \in H$ which contains $f(p_0)$, there is a set $X \in G$ which contains p_0 such that $f[X \cap V] \subseteq Y$, i.e. if $p \in X \cap V$, then $f(p) \in Y$.

1. Let $V = \{1,2,3\}$ and $U = \{a,b,c\}$, and let $G = \{\emptyset, \{1,2\}, V\}$ $H = \{\emptyset, \{a\}, \{a,b\}, U\}$ be topologies on V and U respectively.

   (a). If $f:V \to U$ is defined by $f(1) = a$, $f(2) = a$ and $f(3) = c$, show that f is continuous at each point of V.

   (1). Is f continuous at 1?

   $f(1) = a$ is an element of the following sets in H: $\{a\}$, $\{a,b\}$, $\{a,b,c\}$. Thus, since there exists a set $\{1,2\} \in G$ s.t. $1 \in \{1,2\}$ and $f[\{1,2\}] = \{a\}$, and since $\{a\}$ is a subset of $\{a\}$, $\{a,b\}$ and $\{a,b,c\}$, then f is continuous at 1.

   (2). Is f continuous at 2?

   (3). Is f continuous at 3?

   (b). Let $f:V \to U$ be defined by $f(1) = c$, $f(2) = a$ and $f(3) = b$, show that f is <u>not</u> continuous at the points 2,3 $\in V$.

   iff $\exists\ Y \in H$ which contains $f(p_0)$ s.t. $\forall\ X \in G$ which contains p_0, $\exists\ p \in V \cap X$, but $f(p) \notin Y$.

$\{a\}$	a	$\{1,2\}$	2	1	$f(1) = c \notin \{a\}$
		$\{1,2,3\}$	2	1	$f(1) = c \notin \{a\}$

2. Given the topologies $G = \{\emptyset, \{1,2\}, V\}$ and $H = \{\emptyset, \{1\}, \{1,2\}, V\}$ on the set $V = \{1,2,3\}$, determine if the function $f:V \to V$ defined by $f(1) = f(2) = 2$ and $f(3) = 1$ is continuous at the three points of V.

. Let $V = \{1,2,3\}$ and let $G = \{\emptyset, \{1\}, \{1,2\}, V\}$ be a topology on V. If $f:V \to V$ is defined by $f(1) = 2$, $f(2) = 1$ and $f(3) = 3$, determine if f is continuous at each of the points of V.

4. Let $V = \{1,2,3\}$ and let $G = \{\emptyset, \{2\}, \{1,2\}, V\}$, define two functions $f:V \to V$ such that f is continuous on V.

Def. The function $f:D \to R$ is continuous at point $a \in D$ iff for any real number $\varepsilon > 0$, there exists a real number $\delta > 0$ such that if $x \in D$ and $|x - a| < \delta$, then $|f(x) - f(a)| < \varepsilon$.

The topology is the collection of open intervals on the real number line.

f $\forall\ Y \in H$ which contains $f(p_0)$, $\exists\ X \in G$ which contains p_0 s.t. $\forall\ p \in X \cap V$, then $f(p) \in Y$.

$\forall$ ε-interval of $f(x_0)$, $\exists$ δ-interval of x_0 s.t. $\forall$ x in δ-interval, $f(x)$ in ε-interval.

$f(x_0)-\varepsilon$ $f(x_0)+\varepsilon$ $x_0-\delta$ $x_0+\delta$ $x_0-\delta < x < x_0+\delta$ $f(x_0)-\varepsilon < f(x) < f(x_0)+\varepsilon$

$f(x_0)$ x_0 $|x - x_0| < \delta$ $|f(x) - f(x_0)| < \varepsilon$

5. Show algebraically that $f(x) = 2x$ is continuous at $x = 3$.

   a). For any given epsilon (ε), find a delta (δ), i.e. δ is a function of ε.

   b). Show that if $|x - 3| < \delta$, then $|f(x) - f(3)| < \varepsilon$.

6. Show algebraically that $f(x) = x^2$ is continuous at $x = 2$.

UNIT: ALGEBRAIC STRUCTURES ON SETS

This unit focuses on the algebraic structures on sets and on the technical language that is used in these systems. The topics are binary operations, groups and subgroups, normal and factor groups, and group endomorphisms on finite sets.

The objectives for the students are

To construct operation tables that define an operation on a finite set, that define an operation which satisfies a given property, and that define an operation which satisfies the group properties.

To form magic squares, to construct an operation table of the composition of rotations and/or flips of magic squares, and to identify the subgroups of this system.

To find left and right cosets of a given subgroup, find the factor groups of the normal subgroups, and construct the induced operation tables of the normal subgroups.

To identify functions that are group endomorphisms for a given group, and construct operation tables for addition and composition of the endomorphisms on groups with three elements and groups with four elements.

This unit will be taught in four lessons. In teaching each lesson there will be emphasis on recognizing patterns, using reasoning skills, and interpreting and applying definitions. The attached worksheets will be used to structure, focus and guide the classwork, and to provide homework assignments.

<u>Technical Terms</u>

operation	row
closed	column
group	transformation
subgroup	rotation
coset	flip
associative	composition
identity	endomorphism
inverse	homomorphism
commutative	normal
left-cancellation	factor group
left-identity	function
weakly left-linear	unique
weakly left-solvable	array
left-faithful	main diagonal
magic squares	operation table

UNIT: ALGEBRAIC STRUCTURES ON SETS
Binary Operations

Name ______________________________

Def. The function $+: V \times V \to V$ is a binary operation on the set V iff to each element $(x,y) \in V \times V$, + assigns a unique element of V.

1. Which of the following tables define binary operations on the set V = {a,b,c}?

$+_1$	a	b	c
a	a	a	a
b	a	a	a
c	a	a	a

$+_2$	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

$+_3$	a	b	c
a	a	b	c
b	e	e	e
c	c	a	b

$+_4$	a	b	c
a	a	a	a
b	b	b	b
c	c	c	c

$+_5$	a	b	c
a	a	b	c
b	b		a
c		b	

$+_6$	a	b
a	a	b
b	b	a

2. Define 12 additional binary operations on the set V = {a,b,c}.

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

3. How many binary operations can be defined on a set with three elements? ________

4. Complete each of the following tables so that it defines an operation on the set V = {a,b,c} which satisfies the given property. How are the entries in the table affected by the property?

(a). If $x, y \in V$, then $x + y = y + x$.

+	a	b	c
a	a	c	a
b	c	b	b
c	a	b	c

+	a	b	c
a	c	___	___
b	c	a	b
c	b	___	b

+	a	b	c
a	a	c	___
b	___	a	___
c	b	___	a

+	a	b	c
a			
b			
c			

Commutative- ____

(b). There is an element $e \in V$ such that $e + x = x$ for every $x \in V$.

+	a	b	c
a	b	a	b
b	a	b	c
c	c	a	b

+	a	b	c
a	a	b	b
b	b	b	c
c	___	___	___

+	a	b	c
a	a	___	___
b	___	c	b
c	___	c	___

+	a	b	c
a			
b			
c			

Left-identity- ____

(c). If x is in V, then there is an element $e \in V$ such that $e + x = x$.

+	a	b	c
a	c	b	a
b	a	c	c
c	b	a	a

+	a	b	c
a	b	c	a
b	b	___	___
c	___	c	b

+	a	b	c
a	___	c	b
b	b	a	___
c	b	___	a

+	a	b	c
a			
b			
c			

Local-left-identity- ____

(d). If $x, y, z \in V$ and $x + y = x + z$, then $y = z$.

+	a	b	c
a	a	b	c
b	a	c	b
c	a	b	c

+	a	b	c
a	a	c	___
b	___	c	b
c	b	___	c

+	a	b	c
a	b	___	a
b	c	___	b
c	___	c	___

+	a	b	c
a			
b			
c			

Left-cancellation- ______________________________

(e). If $x, y \in V$, then there is an element $z \in V$ such that $z + x = y$.

+	a	b	c
a	a	c	c
b	c	b	a
c	b	a	b

+	a	b	c
a	___	b	b
b	a	c	___
c	b	___	c

+	a	b	c
a	c	a	___
b	___	c	b
c	a	___	___

+	a	b	c
a			
b			
c			

Weakly-left-solvable- ______________________________

(f). If $x, y, z \in V$, then there is an element $u \in V$ such that $x + y = u + z$.

+	a	b	c
a	c	b	a
b	a	a	b
c	b	c	c

+	a	b	c
a	c	___	a
b	___	a	b
c	b	c	___

+	a	b	c
a	b	c	b
b	___	___	b
c	___	c	c

+	a	b	c
a			
b			
c			

Weakly-left-linear- ______________________________

(g). For an $e \in V$, there is for each $y \in V$ an element $x \in V$ such that $x + y = e$.

+	a	b	c
a	b	c	a
b	a	b	c
c	c	a	b

+	a	b	c
a	___	b	c
b	c	c	___
c	b	___	b

+	a	b	c
a	c	b	___
b	___	___	___
c	___	c	a

+	a	b	c
a			
b			
c			

Left-inverse- ______________________________

(h). If $y, z \in V$ and $x + y = x + z$ for every $x \in V$, then $y = z$.

+	a	b	c
a	a	b	c
b	c	a	b
c	b	c	a

+	a	b	c
a	a	___	a
b	c	___	b
c	b	___	b

+	a	b	c
a	c	___	___
b	___	c	b
c	a	___	a

+	a	b	c
a			
b			
c			

Left-faithful- ______________________________

How many commutative operations can be defined on a set with three elements? ________

How many operations with an identity can be defined on a set with three elements? ______

How many operations with inverses can be defined on a set with three elements? ________

Def. The system $(V,+)$ is a group iff the following properties hold:

i. If $x, y \in V$, then $x + y \in V$.
ii. If $x, y, z \in V$, then $(x + y) + z = x + (y + z)$.
iii. There is an element $e \in V$ such that for every $x \in V$, then $e + x = x + e = x$.
iv. For each $y \in V$, there is an $x \in V$ such that $x + y = y + x = e$, for an $e \in V$.

Complete each of the following tables so that it defines a group.

+	a	b	c
a	___	___	___
b	___	c	___
c	___	___	b

+	a	b	c
a	c	___	___
b	___	___	___
c	___	___	a

+	a	b	c
a	b	___	___
b	___	a	___
c	___	___	___

UNIT: ALGEBRAIC STRUCTURES ON SETS

Groups and Subgroups

Name______________________________

Observe the 3-by-3 squares in Figures 1 and 2 below. You may notice that the sum of numbers in each row, each column and each main diagonal is equal to 15. How are the two squares related?

4	9	2
3	5	7
8	1	6

Fig. 1

8	1	6
3	5	7
4	9	2

Fig. 2

		8
2		6

Fig. 3

6		2
8		

Fig. 4

Def. An array of squares as above is a magic square iff the sums of the numbers in each row, each column, and each main diagonal are equal.

Try your luck at completing the squares in Figures 3 and 4 so that the sum of the numbers in each row, column and main diagonal is 15. How are the squares in Figures 3 and 4 related to the square in Figure 1?

It appears that if we start with a magic square and then perform a series of flips and rotations (transformations) on it, we obtain still another square that is magic.

How many different magic squares can we get from transformations on Figure 1? Identify the kind of transformation which gives each square from Figure 1.

2		
6		8

8		
6		2

6		8
2		

2		6
		8

Further observation tells us that if we *rotate the square in Figure* 1 90° clockwise (R_{90°), then flip the square that results along its verticle axis (F_v), we obtain the square in Figure 3, which is also magic.

Now, let's flip the square in Figure 1 along its verticle axis and then follow the flip with a rotation of 90° clockwise. We obtain the square in Figure 4 which is also magic. But the squares in Figures 3 and 4 are different. Explain.

Show that the operation of composition on rotations and/or flips of magic squares as described above is closed. Complete the operation table below to support your answer.

- I - no rotation or flip of Fig. 1
- R - 90° clockwise rotation of Fig. 1
- R' - 180° clockwise rotation of Fig. 1
- R'' - 270° clockwise rotation of Fig. 1
- H - flip along the horizontal axis of Fig. 1
- V - flip along the vertical axis of Fig. 1
- D - flip along the top to bottom main diagonal of Fig. 1
- D' - flip along the bottom to top main diagonal of Fig. 1

*	I	R	R'	R''	H	V	D	D'
I								
R								
R'								
R''								
H								
V								
D								
D'								

Def. The system (V,+) is a subgroup of the group (U,+) iff $V \subseteq U$ and the + is the binary operation on V.

1. The following tables define subgroups of ({I,R,R',R'',H,V,D,D'}, *).

*	I	R'
I	I	R'
R'	R'	I

*	I	H
I	I	H
H	H	I

*	I	R	R'	R''
I	I	R	R'	R''
R	R	R'	R''	I
R'	R'	R''	I	R
R''	R''	I	R	R'

2. Construct the operation tables for the remaining subgroups of ({I,R,R',R'',H,V,D,D'},

ALGEBRAIC STRUCTURES ON SETS

Normal and Factor Groups

Name________________________________

Given that the table to the right defines an operation on the set G = {I,R,R',R'',H,V,D,D'} such that the system (G,*) is a group.

*	I	R	R'	R''	H	V	D	D'
I	I	R	R'	R''	H	V	D	D'
R	R	R'	R''	I	D	D'	V	H
R'	R'	R''	I	R	V	H	D'	D
R''	R''	I	R	R'	D'	D	H	V
H	H	D'	V	D	I	R'	R''	R
V	V	D	H	D'	R'	I	R	R''
D	D	H	D'	V	R	R''	I	R'
D'	D'	V	D	H	R''	R	R'	I

1. Def. The system (H,+) is a subgroup of the group (G,+) iff $H \subseteq G$ and + is a binary operation on H.

   a. The following tables define subgroups of (G,*).

*	I	R'
I	I	R'
R'	R'	I

*	I	H
I	I	H
H	H	I

*	I	R	R'	R''
I	I	R	R'	R''
R	R	R'	R''	I
R'	R'	R''	I	R
R''	R''	I	R	R'

   b. Construct the operation tables for the remaining subgroups of (G,*).

2. Def. Let H be a subgroup of group G. The set Ha is a right coset of H iff Ha = {ha:h ε H and "a" is a fixed element of G}.

   a. Since I*V = V and R'*V = H, then the right coset of {I,R'} is {I,R'}V = {V,H}.
      Since I*R = R and R'*R = R'', then another right coset of {I,R'} is {I,R'}R = {R,R''}.

   b. List all the left and right cosets of {I,R'}.

Left Cosets	Right Cosets
I {I,R'} =	{I,R'} I =
R {I,R'} =	{I,R'} R =
R'{I,R'} =	{I,R'} R' =
R''{I,R'} =	{I,R'} R'' =
H {I,R'} =	{I,R'} H =
V {I,R'} =	{I,R'} V =
D {I,R'} =	{I,R'} D =
D'{I,R'} =	{I,R'} D' =

3. Def. H is a normal subgroup of G iff for every a ε G, aH = Ha.

   a. Is {I,R'} a normal subgroup of (G,*)?

   b. Complete the operation table below of the cosets of {I,R'}.

*	{I,R'}	{R,R''}	{H,V}	{D,D'}
{I,R'}	{I,R'}	{R,R''}	{H,V}	{D,D'}
{R,R''}	{R,R''}	{I,R'}		
{H,V}	{H,V}			
{D,D'}	{D,D'}			

4. Def. (G/H,*) is a factor group of G modulo H iff H is a normal subgroup of G and the elements of G/H are the cosets of H in G.

2. List all the left and right cosets of {I,H}.

*	I	R	R'	R''	H	V	D	D'
I	I	R	R'	R''	H	V	D	D'
R	R	R'	R''	I	D	D'	V	H
R'	R'	R''	I	R	V	H	D'	D
R''	R''	I	R	R	D'	D	H	V
H	H	D'	V	D	I	R'	R''	R
V	V	D	H	D'	R'	I	R	R''
D	D	H	D'	V	R	R''	I	R'
D'	D'	V	D	H	R''	R	R'	I

Left Cosets	Right Cosets
I {I,H} =	{I,H} I =
R {I,H} =	{I,H} R =
R'{I,H} =	{I,H} R' =
R''{I,H} =	{I,H} R'' =
H {I,H} =	{I,H} H =
V {I,H} =	{I,H} V =
D {I,H} =	{I,H} D =
D'{I,H} =	{I,H} D' =

Is {I,H} a normal subgroup of ({I,R,R',R'',H,V,D,D'}, *)?

Complete the operation table of the right cosets of {I,H}.

*	{I,H}	{R,D}	{R',V}	{R'',D'}
{I,H}				
{R,D}				
{V,R'}				
{R'',D'}				

Compare the operation tables of cosets of subgroups that are normal and subgroups that are not normal. What do you find?

3. Which of the remaining subgroups of ({I,R,R',R'',H,V,D,D'}, *) are normal? List the cosets of each (factor groups).

UNIT: ALGEBRAIC STRUCTURES ON SETS

Group Endomorphisms

Name______________________________

Def. The function $f:V \to V$ is a group endomorphism on $(V,+)$ iff $f(x + y) = f(x) + f(y)$ for all $x,y \in V$.

A. Let $V = \{a,b,c\}$.

1. Show that $f_2:V \to V$ defined by $f_2(a) = a$, $f_2(b) = c$ and $f_2(c) = b$ is a group endomorphism on $(V,+)$ where the operation + is defined by the table $\longrightarrow$

+	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

$$f_2(a + a) = f_2(a) + f_2(a) \qquad f_2(b + a) = f_2(b) + f_2(a) \qquad f_2(c + a) = f_2(c) + f_2(a)$$
$$f_2(a) = a + a \qquad f_2(b) = c + a \qquad f_2(c) = b + a$$
$$a = a \qquad c = c \qquad b = b$$

$$f_2(a + b) = f_2(a) + f_2(b) \qquad f_2(b + b) = f_2(b) + f_2(b) \qquad f_2(c + b) = f_2(c) + f_2(b)$$
$$f_2(b) = a + c \qquad f_2(c) = c + c \qquad f_2(a) = b + c$$

______ ______ ______

$$f_2(a + c) = f_2(a) + f_2(c) \qquad f_2(b + c) = f_2(b) + f_2(c) \qquad f_2(c + c) = f_2(c) + f_2(c)$$

2. Show that $f_1:V \to V$ defined by $f_1(a) = a$, $f_1(b) = b$ and $f_1(c) = c$ is a group endomorphism on $(V,+)$ where + is defined as above.

3. Show that $f_0:V \to V$ defined by $f_0(a) = f_0(b) = f_0(c) = a$ is a group endomorphism on $(V,+)$ where + is defined as above.

4. Construct tables for the following operations on the above group endomorphisms for $x \in V$.

a. $(f_i \oplus f_j)(x) = f_i(x) + f_j(x)$

$\oplus$	f_0	f_1	f_2
f_0			
f_1			
f_2			

b. $(f_i \odot f_j)(x) = f_i(f_j(x))$

$\odot$	f_0	f_1	f_2
f_0			
f_1			
f_2			

B. Let $V = \{a,b,c,d\}$.

1. What are the group endomorphisms on $(V,+)$ where $+$ is defined by

+	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

2. Construct tables for the following operations on the above group endomorphisms.

   a. $(f_i \oplus f_j)(x) = f_i(x) + f_j(x)$ b. $(f_i \odot f_j)(x) = f_i(f_j(x))$

C. The following are the group endomorphisms on $(\{a,b,c,d\}, +)$ where $+$ is defined by

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

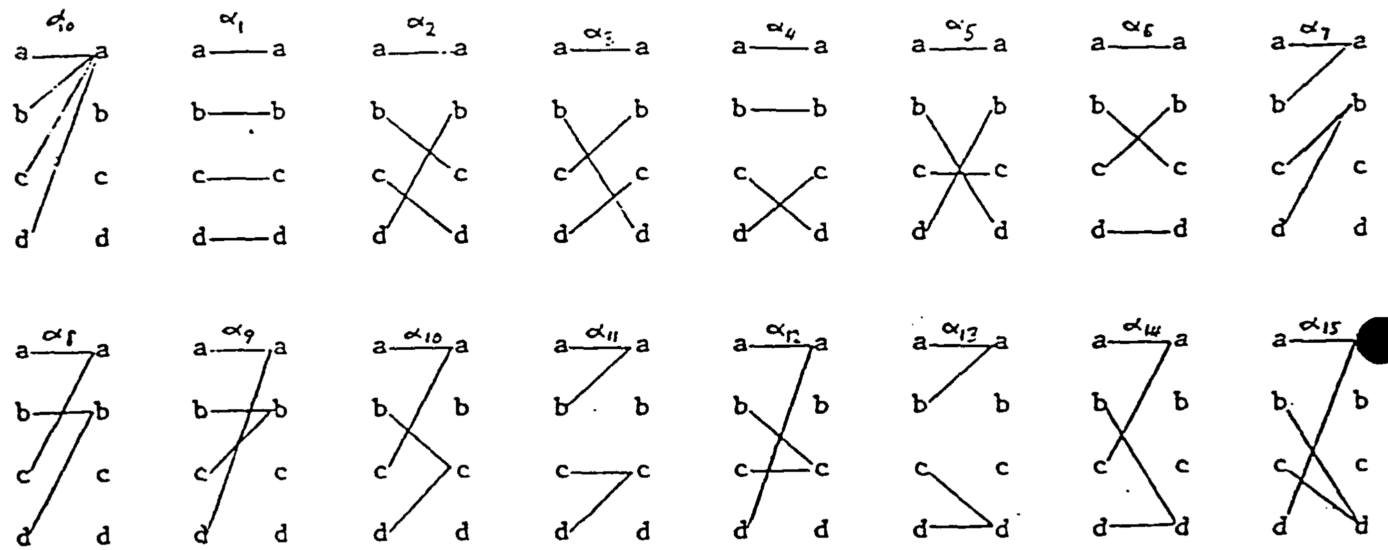
$\{a\}$

$\{a,b\}$

$\{a,c\}$

$\{a,d\}$

$\{a,b,c,d\}$



Construct tables for the following operations on the above group endomorphisms.

a. $(\alpha_i \oplus \alpha_j)(x) = \alpha_i(x) + \alpha_j(x)$ b. $(\alpha_i \odot \alpha_j)(x) = \alpha_i(\alpha_j(x))$

Unit on Groups

Vocabulary

binary operation (closure)
associative law
existence of an identity
existence of inverses
commutative law
group
abelian group
cardinality

Symbols

ϵ (identity); a^{-1} (inverse of a)

Behaviorial Objective

1. Given a Set A, an operation * and an operation table, the student will be able to determine
   a) if Set A is closed under operation *
   b) if Set A is associative under operation *
   c) If there is an identity element of Set A under operation *
   d) the inverses of each element of A under *, if such inverses exist

2. The student will be able to <u>identify</u> commutative and noncommutative groups.

3. The student will cite an example and nonexample of each of the following:
   a) a group
   b) a commutative group
   c) a infinite group

Instructional Strategy

The instructor will use the expository method. Students will be encouraged to focus on relationship and abstract commonalities from examples.

Inverses: The table shows that the set whose members are **a, b** and **c** contains an inverse for every one of its members. The inverse of **a** is **a**, since a combined with **a** is the identity element, i.e. a * a = a. The inverse of **b** is **c**, since **b** combined with **c** is the identity element, i.e. b * c = c * b = a. The inverse of **c** is **b**, since **c** combined with **b** is the identity element, i.e. c * b = b * c = a.

The associative property is satisfied though tedious to show.

Thus we see the the set whose members are a, b, c, taken with the operation *, has all properties that define a group. Therefore, it is a group with respect to operation *. Namely, it is closed, has an identity, each element has an inverse and the associative property is satisfied.

Item 3: The set {-1, 0, 1} with the operation addition

+	-1	0	1
-1	-2	-1	0
0	-1	0	1
1	0	1	2

Closure: A glance at the table shows that -1 + -1 = -2. Since -2 is not an element of the set {-1, 0, 1} , this set is not closed with respect to addition.

Since a defining characteristic is not satisfied the set whose members are -1, 0, 1, taken with operation addition is not a group.

Item 4: The set of whole numbers whose members are 0, 1, 2, 3, 4, ... with operation addition

A partial table is provided:

+	0	1	2	3	.	.	.
0	0	1	2	3	.	.	.
1	1	2	3	4	.	.	.
2	2	3	4	5	.	.	.
3	3	4	5	6	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

Closure: A glance at the table shows that the sum of any two elements of the set results in an element of the set. So the set of whole numbers with operation addition is closed.

Identity: 0 is clearly the identity element because when 0 is added to any element of the set of whole numbers, the element is unchanged, i.e. 0 + 3 = 3 + 0 = 3; 0 + 12 = 12 + 0 = 12.

Inverses: The only element of the set of whole numbers with an inverse under operation addition is 0. The inverse of 0 is 0 because 0 + 0 = 0, the identity. There is no whole number which when

Another way to express all of the products is to use a multiplication table for this set:

x	1	-1
1	1	-1
-1	-1	1

Since each possible product is 1 or -1 and each of these numerals belong to the set {1, -1} we may conclude that the set having members 1 and -1 is closed with respect to multiplication.

Identity - Multiplication of each element of the set by 1 leaves the element unchanged. Therefore, 1 is the identity element in the set whose members are 1 and -1.

Inverses - Examining all of the possible products using elements 1 and -1 we see that every one of these members has an inverse. The inverse of 1 is 1, since 1 x 1 = 1, the identity element, and the inverse of -1, is -1, since -1 x -1 = 1, the identity element.

Associative - Thus we see that the set whose members are 1 and -1, taken with the operation multiplication satisfies the properies of a group. Hence the set {1, -1} is closed, has an identity each element has an inverse and the associative property is satisfied.

Item 1: The set {a,b,c} with the operation * and table:

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

Note: Each element in the table is determined by combining the element in the row and the element in the column. For example, **b** combined with **c** under operation * results in a, i.e. **b * c = a**. Remember (element in row first) * (element in column):

columns

rows

*	a	b	c
a			
b			a
c			

Closure: A glance at table (1) shows that any two elements of the set combined with the operation * results in either **a** or **b** or **c**. That is, the result of the operation * on two members of the set {a,b,c} is also a member of the set {a,b,c}. So the set is closed with respect to operation *.

Identity: Since the result of the operation of either of the elements **a** or **b** or c by **a** leaves it unchanged, **a** is the identity element, i.e.

a * b = b * a = b

a * a = a * a = a

a * c = c * a = c

Inverses: The table shows that the set whose members are **a, b** and **c** contains an inverse for every one of its members. The inverse of **a** is **a**, since **a** combined with **a** is the identity element, i.e. a * a = a. The inverse of **b** is **c**, since **b** combined with **c** is the identity element, i.e. b * c = c * b = a. The inverse of **c** is **b**, since **c** combined with **b** is the identity element, i.e. c * b = b * c = a.

The associative property is satisfied though tedious to show.

Thus we see the the set whose members are a, b, c, taken with the operation *, has all properties that define a group. Therefore, it is a group with respect to operation *. Namely, it is closed, has an identity, each element has an inverse and the associative property is satisfied.

Item 3: The set {-1, 0, 1} with the operation addition

+	-1	0	1
-1	-2	-1	0
0	-1	0	1
1	0	1	2

Closure: A glance at the table shows that -1 + -1 = -2. Since -2 is not an element of the set {-1, 0, 1} , this set is not closed with respect to addition.

Since a defining characteristic is not satisfied the set whose members are -1, 0, 1, taken with operation addition is not a group.

Item 4: The set of whole numbers whose members are 0, 1, 2, 3, 4, ... with operation addition

A partial table is provided:

+	0	1	2	3	.	.	.
0	0	1	2	3	.	.	.
1	1	2	3	4	.	.	.
2	2	3	4	5	.	.	.
3	3	4	5	6	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

Closure: A glance at the table shows that the sum of any two elements of the set results in an element of the set. So the set of whole numbers with operation addition is closed.

Identity: 0 is clearly the identity element because when 0 is added to any element of the set of whole numbers, the element is unchanged, i.e. 0 + 3 = 3 + 0 = 3; 0 + 12 = 12 + 0 = 12.

Inverses: The only element of the set of whole numbers with an inverse under operation addition is 0. The inverse of 0 is 0 because 0 + 0 = 0, the identity. There is no whole number which when added to the other whole numbers, 1, 2, 3, 4, ..., will result in the sum of 0, the identity element with respect to addition in this set. So, not every element in the set of whole numbers has a inverse under the operation addition.

Hence, the set of whole numbers whose members are 0, 1, 2, 3, ... taken with the operation addition is not a group because the inverse property is not satisfied.

WORKSHEET I

NAME______________________________ DATE______________

Directions: Identify each of the following items as an example or nonexample of a group by checking the appropriate box. If the set with the operation is not a group (nonexample), give at least one defining characteristic that is lacking, i.e. not closed, no identity, not every element has an inverse, not associative.

1. The set {-1, 0, 1} with the operation multiplication

x	-1	0	1
-1	1	0	-1
0	0	0	0
1	-0	0	1

example /__/
nonexample /__/ ______________________________

2. The set {0, 1} with operation multiplication

x	0	1
0	0	0
1	0	1

example /__/
nonexample /__/ ______________________________

3. The set {-1, 2} with the operation addition

+	-1	2
-1	-2	1
2	1	4

example /__/
nonexample /__/ ______________________________

4. The set {1} with the operation multiplication

x	1
1	1

example /__/
nonexample /__/ ______________________________

5. The set {-1} with the operation multiplication

x	-1
-1	1

example /__/
nonexample /__/ ______________________________

6. The set {x, y} with the operation *

*	x	y
x	x	y
y	y	z

example /__/
nonexample /__/ ____

7. The set {0,1,2} with the operation *

*	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

example /__/
nonexample /__/ ____

8. The set {-2,0,2} with the operation addition

+	-2	0	2
-2	-4	-2	0
0	-2	0	2
2	0	2	4

example /__/
nonexample /__/ ____

9. The set of whole numbers whose members are 0,1,2,3,... with the operation multiplication

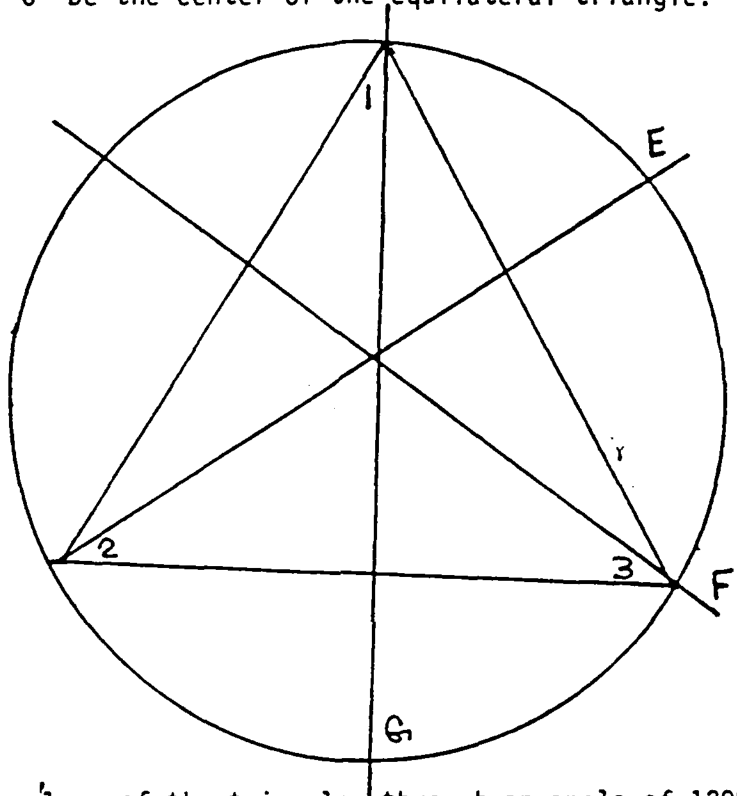
example /__/
nonexample /__/ ____

10. The set of even whole numbers whose members are 0, 2, 4, 6, . . ., with the operation multiplication

example /__/
nonexample /__/ ____

Rigid Motions of a Triangle

Today we will study all rigid motions of a triangle into itself. That is, we will consider motions such that the figure will look the same after the motion as before. Let us designate the vertices of the triangle as 1, 2 and 3. Let E, F and G be axis bisecting the sides of the equilateral triangle and let 0 be the center of the equilateral triangle.



A rotation, in the plane of the triangle, through an angle of 120° counterclockwise about the point 0 would place the vertices in the position 3 1 2. We may interpret the results of this rotation as mapping 1 into 3, 2 into 1 and 3 into 2.

A rigid motion of 120° counterclockwise results in the permutation

$$\alpha_1 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Similary a rigid motion of 240° counterclockwise results in the permutation

$$\alpha_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

A rigid motion of 360°

$$\alpha_3 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

Let α_4 be the permutation which arises from a rotation through an angle of 180^0 about the line E (flip over E); α_5 the permuation arising from a flip over the line F and α_6 the permutaiton arising from a flip over the line G

$$\alpha_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \qquad \text{flip over E}$$

$$\alpha_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \qquad \text{flip over F}$$

$$\alpha_6 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \qquad \text{flip over G}$$

Let us consider the set $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$ of permutations obtained by rigid motions of the triangle and the operation "multiplication." Two permutaticns $\alpha_i\ \alpha_j$ may be multiplied by performing rigid motion α_i followed by rigid motion α_j on the result.

*	α_1	α_2	α_3	α_4	α_5	α_6
α_1			α_1			
α_2			α_2			
α_3			α_3			
α_4			α_4			
α_5			α_5			
α_6	α_5	α_4	α_6	α_2	α_1	α_3

1. Complete the above table.

2. Is the set of permutations closed under operation * ?

3. Is there an identity element? If so name it_______________ .

4. Give the following inverses if they exist.

α_1^{-1} = _______________

α_2^{-1} = _______________

α_3^{-1} = _______________

α_4^{-1} = _______________

α_5^{-1} = _______________

α_6^{-1} = _______________

5. Is the associative property satisfied? _________________
Show two cases to support your answer.

6. Is the set permutations obtained by rigid motions of the triangle with the operation "multiplication" a group?

7. Is it abelian? __________. Show two cases to support your answer.

WORKSHEET III

NAME______________________________ DATE_______________

UNIT: ALGEBRAIC STRUCTURES ON SETS

Groups and Subgroups

Name________________________

Observe the 3-by-3 squares in Figures 1 and 2 below. You may notice that the sum of numbers in each row, each column and each main diagonal is equal to 15. How are the two squares related?

4	9	2
3	5	7
8	1	6

Fig. 1

8	1	6
3	5	7
4	9	2

Fig. 2

		8
2		6

Fig. 3

6		2
8		

Fig. 4

Def. An array of squares as above is a magic square iff the sums of the numbers in each row, each column, and each main diagonal are equal.

Try your luck at completing the squares in Figures 3 and 4 so that the sum of the numbers in each row, column and main diagonal is 15. How are the squares in Figures 3 and 4 related to the square in Figure 1?

It appears that if we start with a magic square and then perform a series of flips and rotations (transformations) on it, we obtain still another square that is magic.

How many different magic squares can we get from transformations on Figure 1? Identify the kind of transformation which gives each square from Figure 1.

2		
6		8

8		
6		2

6		8
2		

2		6
		8

Further observation tells us that if we rotate the square in Figure 1 90° clockwise ($R_{90^{\circ}}$), then flip the square that results along its verticle axis (F_v), we obtain the square in Figure 3, which is also magic.

Now, let's flip the square in Figure 1 along its verticle axis and then follow the flip with a rotation of 90° clockwise. We obtain the square in Figure 4 which is also magic. But the squares in Figures 3 and 4 are different. Explain.

Show that the operation of composition on rotations and/or flips of magic squares as described above is closed. Complete the operation table below to support your answer.

I - no rotation or flip of Fig. 1
R - 90° clockwise rotation of Fig. 1
R' - 180° clockwise rotation of Fig. 1
R'' - 270° clockwise rotation of Fig. 1
H - flip along the horizontal axis of Fig. 1
V - flip along the vertical axis of Fig. 1
D - flip along the top to bottom main diagonal of Fig. 1
D' - flip along the bottom to top main diagonal of Fig. 1

*	I	R	R'	R''	H	V	D	D'
I								
R								
R'								
R''								
H								
V								
D								
D'								

Def. The system (V,+) is a subgroup of the group (U,+) iff $V \subseteq U$ and the + is the binary operation on V.

1. The following tables define subgroups of ({I,R,R',R'',H,V,D,D'}, *).

*	I	R'
I	I	R'
R'	R'	I

*	I	H
I	I	H
H	H	I

*	I	R	R'	R''
I	I	R	R'	R''
R	R	R'	R''	I
R'	R'	R''	I	R
R''	R''	I	R	R'

2. Construct the operation tables for the remaining subgroups of ({I,R,R',R'',H,V,D,D'}, *)

Worksheet IV

Name ______________________________ Date ____________________

Suppose that for any integer we consider only the remainder resulting from division by 5, and we define two integers to be "equivalent" if they have the same remainder. We express that 12 and 17 both have the same remainder when divided by 5 by writing

$$12 \equiv 17 \pmod{5}$$

where $\equiv$ denotes "equivalent" and "mod" is an abbreviation for "modulo". Similarly, we write

$$3 \equiv 8 \pmod{5}$$

1. Consider the set $A = \{0,1,2,3,4\}$ and the binary operation "addition modulo 5", denoted by $\oplus$. Please note that if a and b are two elements of set A, then

$$a \oplus b = 2 \quad \text{if} \quad a + b \equiv 2 \pmod{5}$$

Show that the set A with binary operation "addition modulo 5" constitutes a group. Fill in the chart first.

$\oplus$	0	1	2	3	4
0					
1					
2					
3					
4					

2. Show that the set of integers $\{1,2,3,4\}$ with the binary operation "multiplication modulo 5" is a group.

$\otimes$	1	2	3	4
1				
2				
3				
4				

3. Let $p > 1$ be a prime number, i.e., a number with precisely two positive integral divisors, 1 and p, and consider the set

$$\{1, 2, 3, \ldots, p-1\}$$

We claim that "multiplication modulo p" is a binary operation on this set. Show that the group properties are satisfied.

UNIT ON FUNCTIONS

Vocabulary

Reasoning skills 2-4 will be stressed

set
ordered pair
Cartesian product
relations
functions
inverse of a function
domain of a function
range of a function
"into"
"onto"
1-1
constant function
real-valued function
linear function
independent variable
dependent variable
slope of a line
intercept of a line
identity function
graph of a function

Symbols

ϵ , f , f:A $\rightarrow$ B, A $\xrightarrow{f}$ B, f(a) , a ϵ A

Behavioral Objectives

1. Given two finite sets A and B the student will identify subsets of the Cartesian product which are
   (a) functions of A into B
   (b) functions of A onto B
   (c) functions of A into B which are 1-1
   (d) relations
   (e) constant functions

2. Given a collection of ordered pairs that represent a function, the student will be able to
   (a) identify the domain of the function
   (b) identify the range of the function
   (c) give the inverse of the function
   (d) state whether or not the inverse of the function is itself a function and justify the answer.

3. Given a table of three intergral values of a linear function the student will be able to
   (a) write a corresponding equation of the function
   (b) give the slope of the line determined by the function
   (c) sketch the graph of the function
   (d) give the x- and y- intercept of the line

4. Given several graphs of relations, the student will be able to identify those graphs which represent functions.

5. Given information regarding the slope of a line, i.e., positive slope, negative slope, slope of '0', slope does not exist, the student will be able to discuss the direction of the corresponding line.

6. Given a table of three integral values of a quadratic function, the student will be able to
   a) write a corresponding equation of the function

7. Given at least five terms of a sequence, the student will find the next number and the n^{th} term.

Scanigs

NAME ______________________

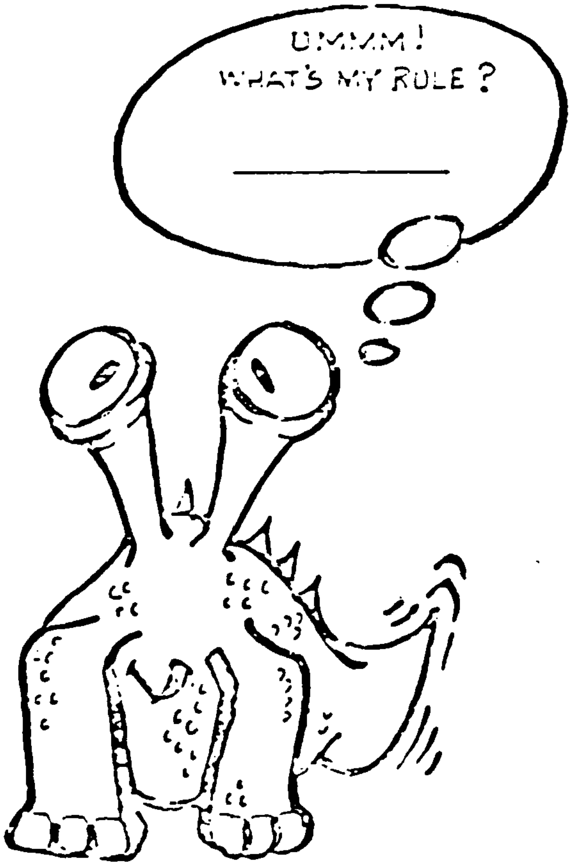
All of these are scanigs.

21 → 35   71 → 85

41 → 55   22 → 36

Make scanigs out of these:

1. 3 → ☐
2. 19 → ☐
3. 47 → ☐
4. 55 → ☐
5. 0 → ☐
6. 108 → ☐



Make your own scanigs!

◯ → ☐

◯ → ☐

1. The Cartesian product of sets A and B is the set of all ordered pairs such that the first element of each pair is an element of A and the second element of each pair is an element of B (written $A \times B$).

   If $A = \{a,b,c\}$ and $B = \{1,2\}$, list the elements of $A \times B$ and the elements of $B \times A$.

   Also, list the elements of $A \times A$.

2. A relation from set A into set B is a subset of $A \times B$.

   If $A = \{a,b,c\}$ and $B = \{1,2\}$, define five relations from set A into set B.

   How many relations are there from set A into set B?

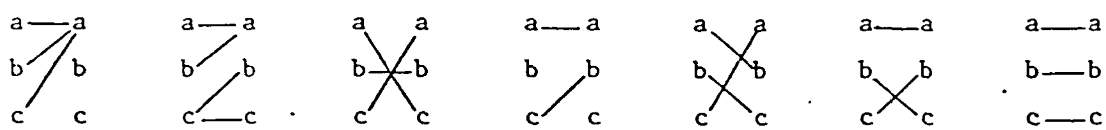
   How many relations are there from set A into set A?

3. A function from set A into set B is a relation from A into B in which each element of A is paired with one and only one element of B.

   i. The set of all first components of a function, all the elements of A, is called the domain of the function.

   ii. The set of all second components of a function, a subset of B, is called the range of the function.

   Which of the following relations define functions on the set $A = \{a,b,c\}$?



   What is the range of each of the functions?

   How many functions are there from set A into set A?

   If $A = \{a,b,c\}$ and $B = \{1,2\}$, how many functions are there from set A into set B?

   If set A has n elements and set B has m elements, determine the following:

   a. How many relations are there from set A into set B?

   b. How many functions are there from set A into set B?

WORKSHEET I

NAME____ DATE________________

1. If A = {1,2,3 }and B = {a,b,c } the Cartesian product of A and B, denoted AXB, is

{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c), (3,a), (3,b), (3,c)}

a. Give a subset of AXB which represents a function from A into B.

____

b. Give a different subset of AXB which represents a function from A onto B.

____

c. Give a different subset a AXB which represents a 1-1 function from A into B.

____

d. Give a different subset of AXB which represents a relation which is not a function.

____

e. Give a different subset of AXB which represents a function from A into B which is a constant function.

____

f. Give a different subset of AXB which represents a function from A into B which is not onto.

____

2. Let A = {1,2,3,...20 } and B = { 1,2,3,...100 } and $f:A \rightarrow B$ such that $f(a) = 2a$ for every $a \in A$.

a. Give the other elements of f{(1,2) ______________}

b. Give the domain of f.________________

c. Is f onto? __________________

d. Is f 1-1? __________________

e. Give the inverse of f, denoted f^{-1}

______________________________ ________________

f. Give the domain of f^{-1}

____

g. Give the range of f^{-1}

____

h. Is f^{-1} a function?

____

Justify your answer.

____

____

WORKSHEET II
"GUESSING FUNCTIONS"

NAME____ DATE_______________

Find the rule associated with each integer chart and complete the accompanying statement:

(1)

X	f(x)
1	6
2	11
3	16

Rule: $f(x) = 5x + 1$
For every unit increase in x,
there is a 5 unit increase in f(x)

(2)

Δ	$\square$
2	1
3	0
4	-1

Rule:________________
For every unit increase
in Δ , there is a ____
unit ____________ in $\square$.

(3)

x	y
3	13
4	15
5	17

Rule:____________________
For every unit increase in x,
there is a ______ unit ________ in y.

(4)

z	f(z)
2	0
4	1
5	3/2

Rule:____________________
For every unit increase in
z, there is a ______ unit
____________ in f(z).

(5)

Δ	$\square$
2	-3
3	-7
4	-11

Rule:________________________
For every unit increase in Δ ,
there is a ______ unit __________ in $\square$.

(6)

x	f(x)
3	-5
5	-7
7	3/2

Rule:____________________
For every unit increase in x,
there is a _____ unit__________
in f(x).

(7)

x	f(x)
1	-4/7
3	-2
4	-2 5/7

Rule:________________________
For every unit increase in x,
there is a ______ unit ________ in f(x)

(8)

x	y
11	0
15	4
25	14

Rule:____________________
For every unit increase in x,
there is a ______ unit ________in y.

(9)

z	f(z)
0	10
10	14
20	18

Rule:______________________
For every unit increase in z,
there is a _______ unit _______ in f(z)

(10)

z	f(z)
2	3
10	3
15	3

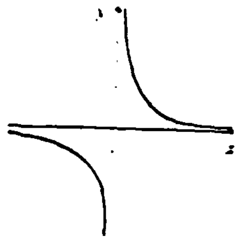
Rule:____________________
For every unit increase in z,
there is a _______ unit_______
in f(z).

WORKSHEET III

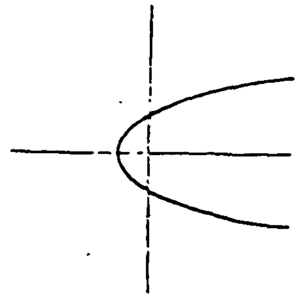
1. Sketch the graph of each rule given on Worksheet II, and note the slope and intercepts.

2. Which of the following graphs represent functions. Write yes or no and justify.

(a)

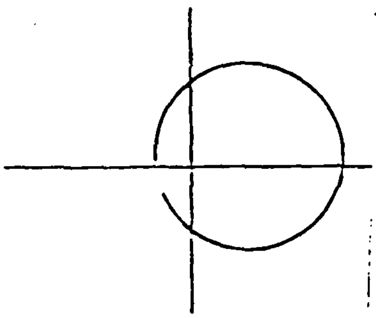


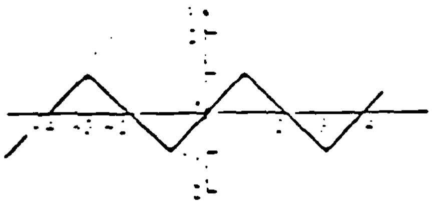
(b)



(a)____________________________

(b)____________________________





(c)____________________________

(d)____________________________

3. Discuss the direction of the line if the slope of that line
   a) is negative

   ____

   ____

   ____

   b) is positive

   ____

   ____

   ____

   c) is 0

   ____

   ____

   ____

   d) does not exist

   ____

   ____

   ____

WORKSHEET IV

A SEQUENCE IS A FUNCTION

NAME____ DATE_______________________

NUMBER SEQUENCES

A number sequence is a succession of numbers arranged according to some definite pattern. Any number in this sequence is related to its preceding number according to a definite plan. In the relatively simple sequence, 3, 5, 7, 9, 11 .... two is added to each number in the sequence. Other relationships may involve the processes of subtraction, multiplication, division, squaring, extracting roots and, in the case of more difficult problems, it may involve a combination of these processes. For example, in the sequence 1, 2, 2, 5, 3, 10, 4, 17, 5, the second number in each pair is the square of the first number plus 1. Thus, 2 is $1^2 + 1$, 5 is $2^2 + 1$, 10 is $3^2 + 1$, etc. Similarly, in the sequence 3, 13, 53, 213 .... one is added to the product of the preceding number and 4.

PRACTICE EXERCISES

Find the next number in each of the following sequences

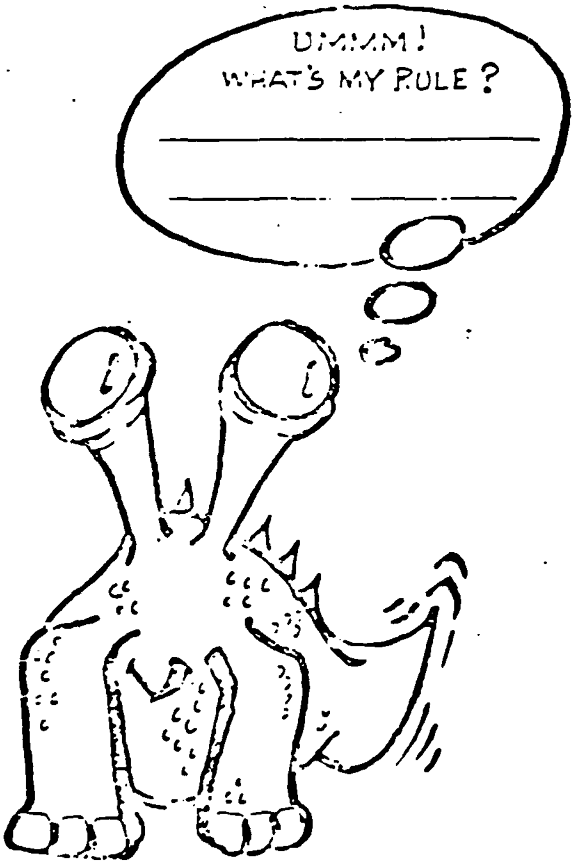
1. 1, 3, 5, 7, 9 . . .
2. 3, 3, 6, 6, 9, 9, 12 . . .
3. 6, 11, 16, 21, 26 . . .
4. −9, −6, −3, 0 . . .
5. 36, 39, 39, 43, 43, 48, 48 . . .
6. $28\frac{1}{8}, 21\frac{1}{8}, 14\frac{1}{8}, 7\frac{1}{8}$ . . .
7. 5, 7, 11, 17, 25 . . .
8. −26, −20, −14, −8 . . .
9. 18, 26, 27, 35, 36, 44 . . .
10. 5, 6, 8, 11, 15, . . .
11. $7\frac{1}{2}, 6\frac{3}{4}, 6, 5\frac{1}{4}, 4\frac{1}{2}$ . . .
12. $9\frac{3}{4}, 9\frac{5}{8}, 9\frac{1}{2}, 9\frac{3}{8}$ . . .
13. 2.52, 3.02, 3.52, 4.02 . . .
14. 5.3, 6.4, 7.5, 8.6, 9.7 . . .
15. 11, 28, 79, 232 . . .
16. 5, 8, 9, 12, 13, 16 . . .
17. 7, 8, 10, 13, 17, 22 . . .
18. 79, 77, 75, 73, 71 . . .
19. 79, 74, 70, 67, 65 . . .
20. 21, 20, 18, 15, 11, 6 . . .
21. 13, 12, 10, 7 . . .
22. −5, 10, −20, 40, −80 . . .
23. −2, −4, −8, −16, −32 . . .
24. 1, 4, 9, 16, 25, 36, 49 . . .
25. 5, 10, 17, 26, 37 . . .
26. 2, 6, 14, 30, 62 . . .
27. 1782, 594, 198, 66 . . .
28. 4752, 792, 132 . . .
29. 99, 88, 77, 66, 55 . . .
30. 100, 81, 64, 49, 36 . . .
31. 5, 2, 5, 4, 5, 6, 5 . . .
32. 48, 24, 12, 6 . . .
33. 80, 2, 40, 2, 20, 2 . . .
34. 5, 8, 24, 27, 81, 84 . . .
35. 3, 5, 10, 12, 24, 26 . . .
36. 5, 6, 8, 11, 15, 20 . . .
37. 7, 14, 14, 21, 21, 28, 28 . . .
38. 3, 6, 5, 8, 7, 10, 9 . . .
39. 6, 10, 13, 17, 20, 24 . . .
40. 12, 16, 13, 17, 14, 18 . . .
41. 5, 6, 8, 11, 15, 20, 26 . . .
42. $\frac{1}{15}$, 0.2, 0.6, 1.8 . . .
43. 65, 60, 56, 53, 51 . . .
44. 256, 16, 4, 2 . . .
45. 400, 361, 324, 289, 256 . . .
46. 121, 144, 169, 196 . . .
47. 512, 343, 216, 125, 64 . . .
48. 2, 5, 15, 18, 54 . . .
49. 16, 256, 15, 225, 14, 196, 13 . . .
50. 4, 64, 5, 125, 6 . . .

NAME ____________________

All of these are escams.

3 → 4 9 → 64 4 → 9

5 → 16 21 → 400



Make escams out of these:

1. 6 → [ ]
2. 1 → [ ]
3. 10 → [ ]
4. 2 → [ ]
5. 12 → [ ]
6. 20 → [ ]

Make your own escams!

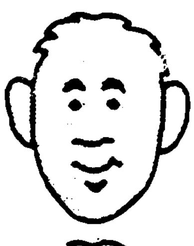
( ) → [ ]

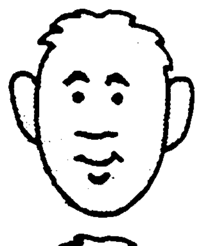
( ) → [ ]

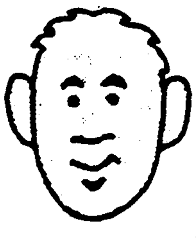
20 →  → 7

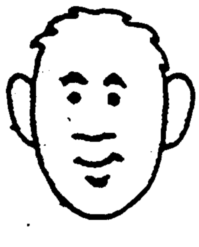
13 →  → 0

23 →  → 10

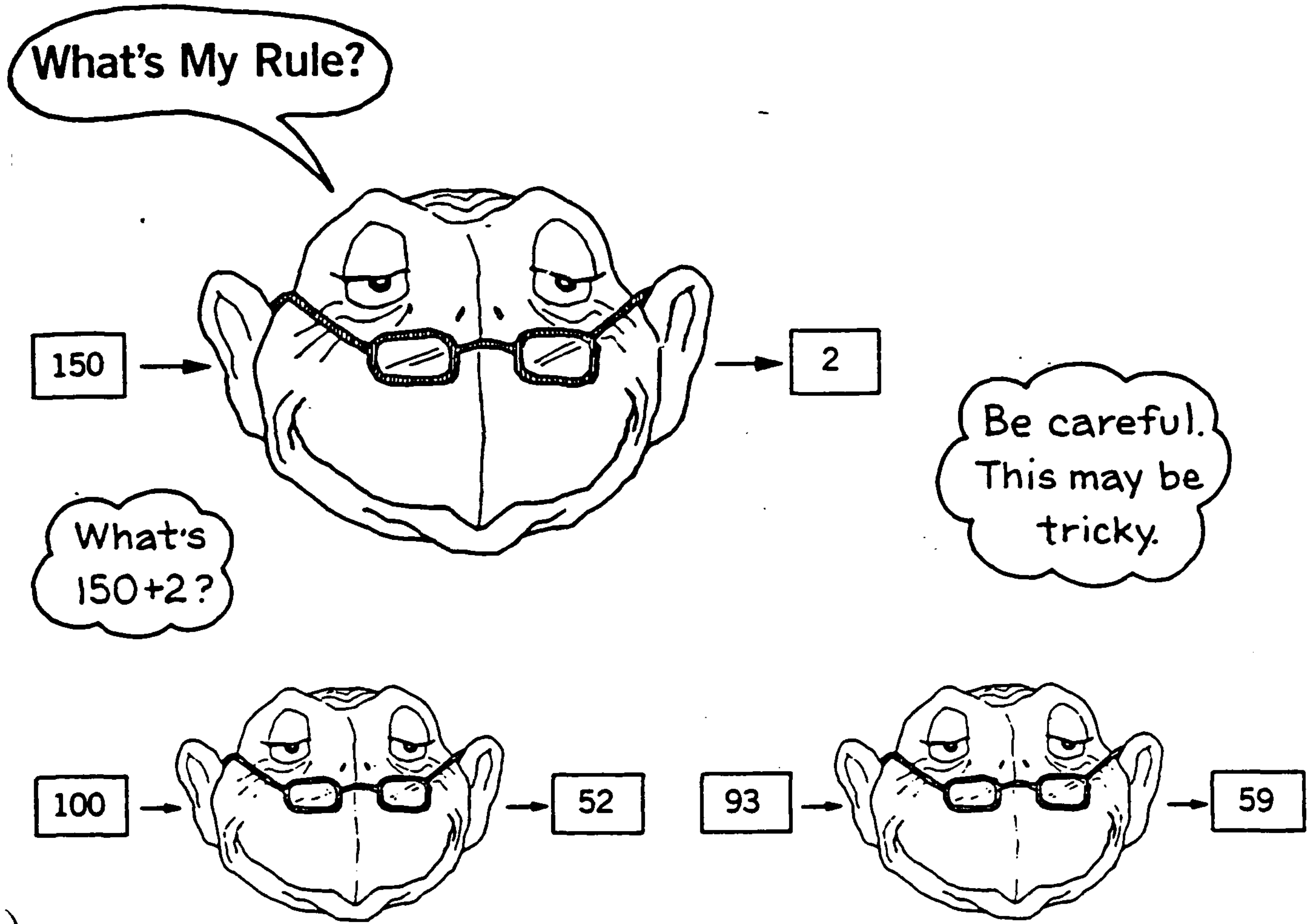
1. 14 →  → ☐

2. 20 →  → ☐

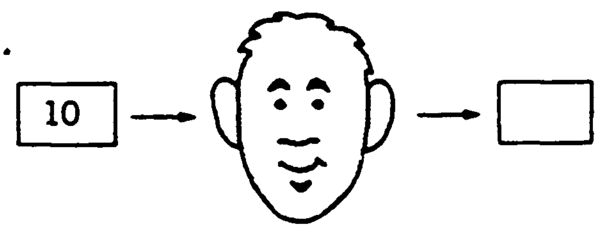
3. 100 →  → ☐

4. 992 →  → ☐

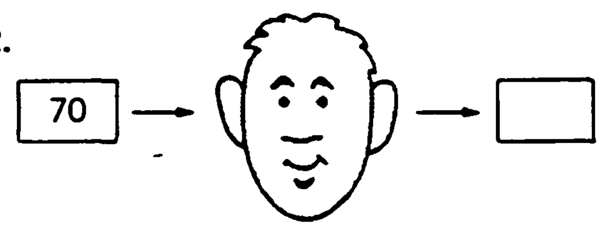
The answer to 4 is the height in meters of the world's highest waterfall.



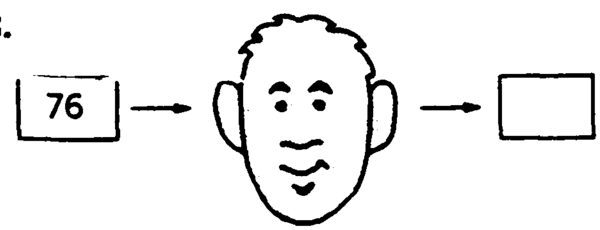
1.



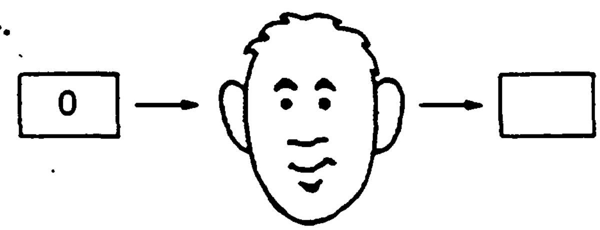
2.



3.



4.



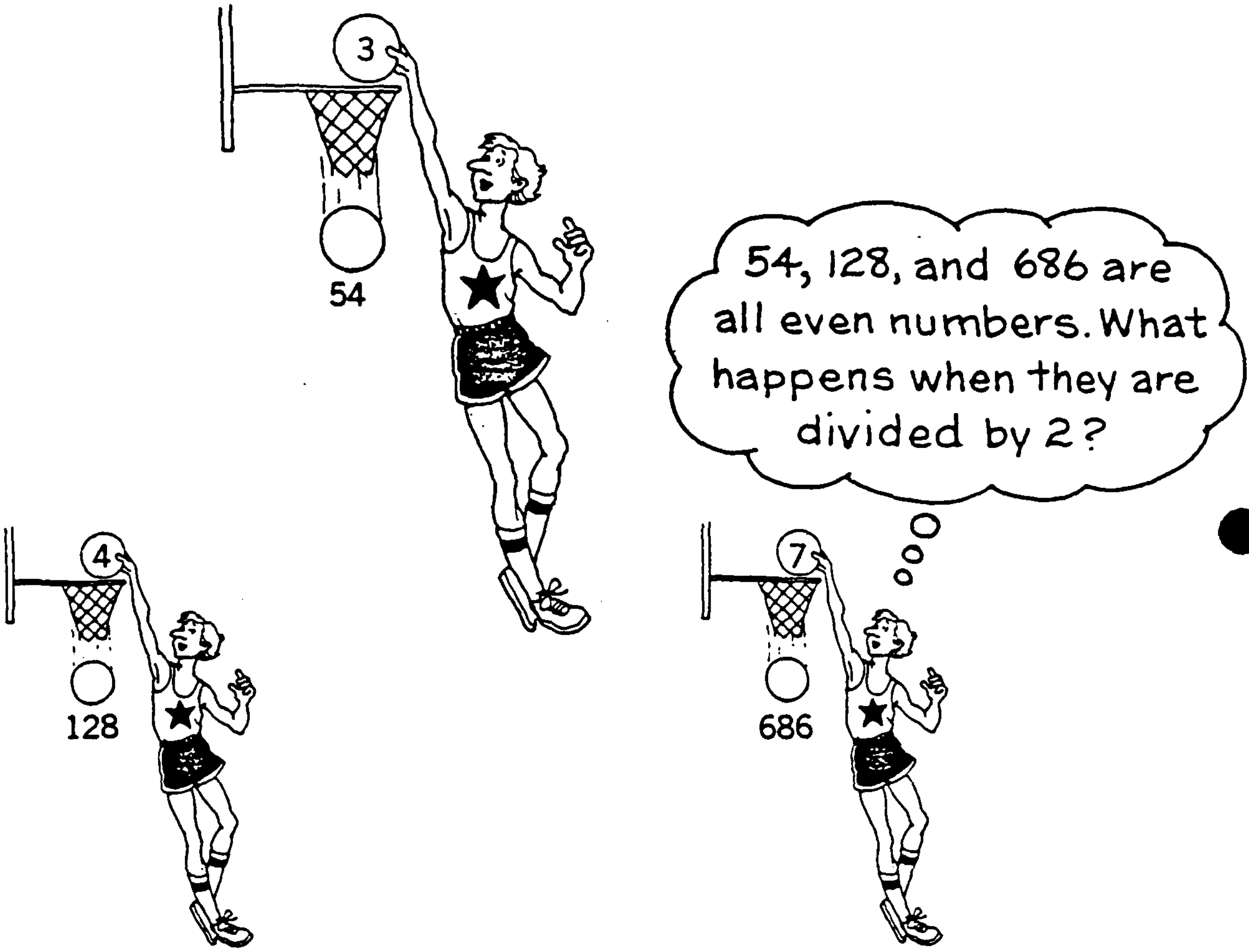
The answer to **4** is the oldest recorded age of any animal (a tortoise).

What's My Rule?

1. 3→□→39, 8→□→104, 11→□→143, 100→□→ ______
2. 5→□→13, 11→□→ 19, 37→□→ 45, 1000→□→ ______
3. 9→□→ 6, 14→□→ 11, 98→□→ 95, 800→□→ ______
4. 5→□→14, 12→□→ 7, 2→□→ 17, 19→□→ ______
5. 3→□→13, 9→□→ 37, 15→□→ 61, 100→□→ ______
6. 1→□→10, 6→□→ 0, 4→□→ 4, 5→□→ ______
7. 4→□→17, 8→□→ 37, 17→□→ 82, 400→□→ ______
8. 3→□→21, 13→□→ 91, 23→□→161, 103→□→ ______
9. 8→□→46, 2→□→ 10, 14→□→ 82, 50→□→ ______
10. 3→□→34, 18→□→ 19, 23→□→ 14, 30→□→ ______
11. 5→□→14, 8→□→ 23, 10→□→ 29, 1000→□→ ______
12. 1→□→87, 7→□→ 69, 15→□→ 45, 20→□→ ______
13. 2→□→29, 8→□→ 77, 10→□→ 93, 500→□→ ______
14. 0→□→58, 9→□→ 49, 45→□→ 13, 50→□→ ______
15. 5→□→15, 2→□→ 9, 17→□→ 39, 500→□→ ______
16. 4→□→ 3, 11→□→ 31, 20→□→ 67, 1010→□→ ______

To check your answers, use the Answer List for Review Problems.

What Comes Out?



1. In: 1

   Out: ☐

2. In: 5

   Out: ☐

3. In: 21

   Out: ☐

4. In: n

   Out: ☐

The answer to 4 is an expression involving n.

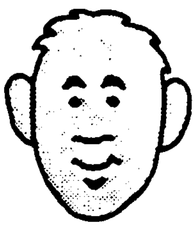
What's My Rule?

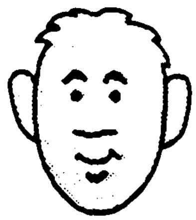
0 →  → 24

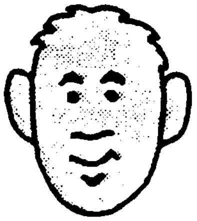
What's special about the "in" numbers here?

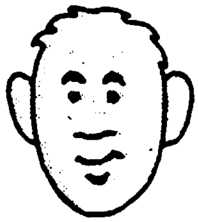
1 →  → 21

2 →  → 18

1. 3 →  → ☐

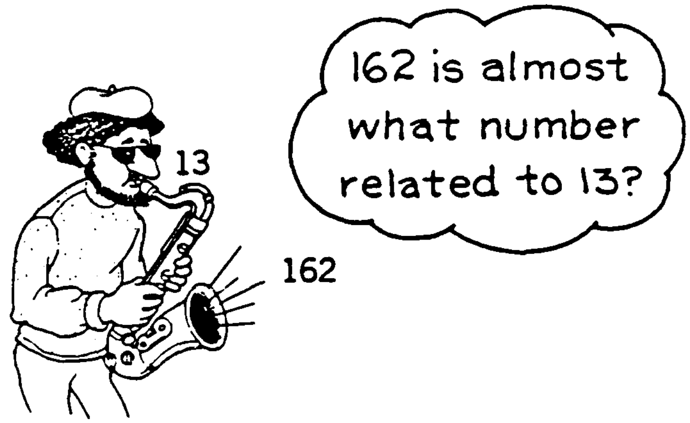
2. 8 →  → ☐

3. 5 →  → ☐

4. 6 →  → ☐

The answer to **4** is equal to both the sum and the product of the same three numbers.

What Comes Out?



1. In: 5

   Out: ______

2. In: 10

   Out: ______

3. In: 17

   Out: ______

4. In: n

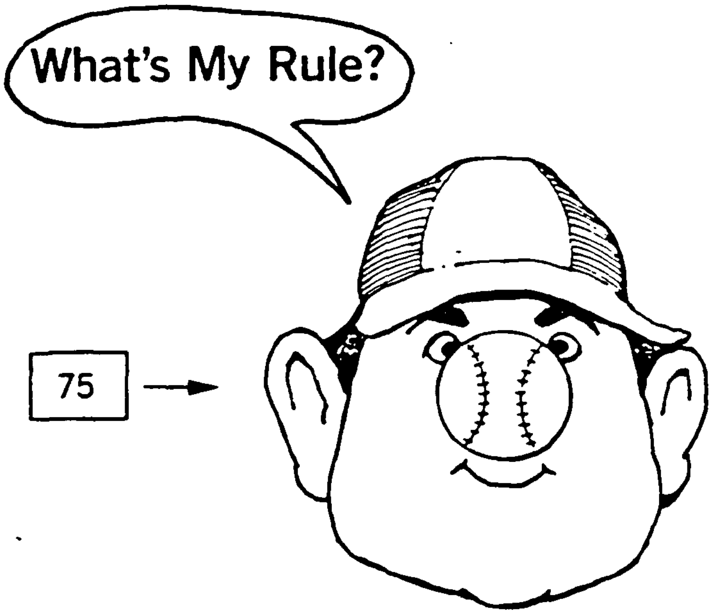
   Out: ______

The answer to 4 is an expression involving n.

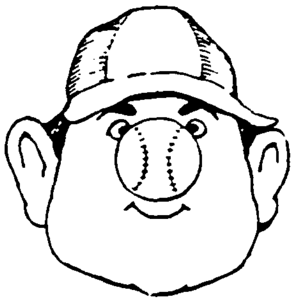
What Comes Out?

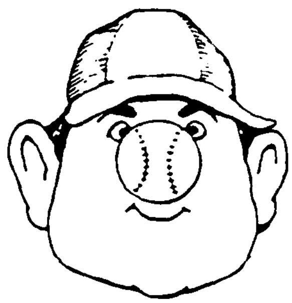
1.	$0 \rightarrow \square \rightarrow 16$,	$16 \rightarrow \square \rightarrow 32$,	$8 \rightarrow \square \rightarrow 24$,	$n \rightarrow \square \rightarrow$	______
2.	$0 \rightarrow \square \rightarrow 0$,	$1 \rightarrow \square \rightarrow 1$,	$8 \rightarrow \square \rightarrow 64$,	$n \rightarrow \square \rightarrow$	______
3.	$10 \rightarrow \square \rightarrow 1$,	$25 \rightarrow \square \rightarrow 16$,	$18 \rightarrow \square \rightarrow 9$,	$n \rightarrow \square \rightarrow$	______
4.	$1 \rightarrow \square \rightarrow 10$,	$2 \rightarrow \square \rightarrow 13$,	$3 \rightarrow \square \rightarrow 18$,	$n \rightarrow \square \rightarrow$	______
5.	$12 \rightarrow \square \rightarrow 60$,	$36 \rightarrow \square \rightarrow 36$,	$4 \rightarrow \square \rightarrow 68$,	$n \rightarrow \square \rightarrow$	______
6.	$8 \rightarrow \square \rightarrow 8$,	$7 \rightarrow \square \rightarrow 23$,	$6 \rightarrow \square \rightarrow 36$,	$n \rightarrow \square \rightarrow$	______
7.	$8 \rightarrow \square \rightarrow 104$,	$7 \rightarrow \square \rightarrow 91$,	$0 \rightarrow \square \rightarrow 0$,	$n \rightarrow \square \rightarrow$	______
8.	$5 \rightarrow \square \rightarrow 125$,	$2 \rightarrow \square \rightarrow 8$,	$3 \rightarrow \square \rightarrow 27$,	$n \rightarrow \square \rightarrow$	______
9.	$5 \rightarrow \square \rightarrow 27$,	$2 \rightarrow \square \rightarrow 12$,	$3 \rightarrow \square \rightarrow 17$,	$n \rightarrow \square \rightarrow$	______
10.	$1 \rightarrow \square \rightarrow 3$,	$4 \rightarrow \square \rightarrow 66$,	$10 \rightarrow \square \rightarrow 1002$,	$n \rightarrow \square \rightarrow$	______
11.	$1 \rightarrow \square \rightarrow 4$,	$4 \rightarrow \square \rightarrow 25$,	$9 \rightarrow \square \rightarrow 60$,	$n \rightarrow \square \rightarrow$	______
12.	$2 \rightarrow \square \rightarrow 5$,	$3 \rightarrow \square \rightarrow 24$,	$7 \rightarrow \square \rightarrow 340$,	$n \rightarrow \square \rightarrow$	______
13.	$3 \rightarrow \square \rightarrow 91$,	$30 \rightarrow \square \rightarrow 10$,	$12 \rightarrow \square \rightarrow 64$,	$n \rightarrow \square \rightarrow$	______
14.	$3 \rightarrow \square \rightarrow 9$,	$1 \rightarrow \square \rightarrow 3$,	$2 \rightarrow \square \rightarrow 5$,	$n \rightarrow \square \rightarrow$	______
15.	$3 \rightarrow \square \rightarrow 26$,	$1 \rightarrow \square \rightarrow 2$,	$2 \rightarrow \square \rightarrow 8$,	$n \rightarrow \square \rightarrow$	______
16.	$6 \rightarrow \square \rightarrow 36$,	$5 \rightarrow \square \rightarrow 68$,	$3 \rightarrow \square \rightarrow 92$,	$n \rightarrow \square \rightarrow$	______

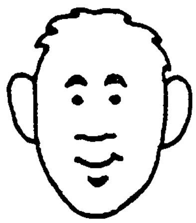
To check your answers, use the Answer List for Review Problems.

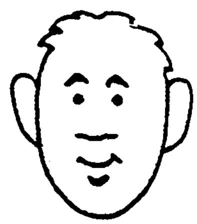


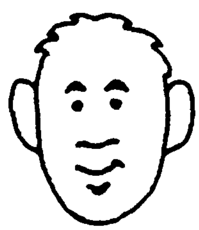
75 → → 50

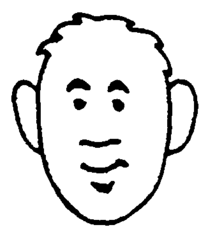
31 →  → 6

92 →  → 67

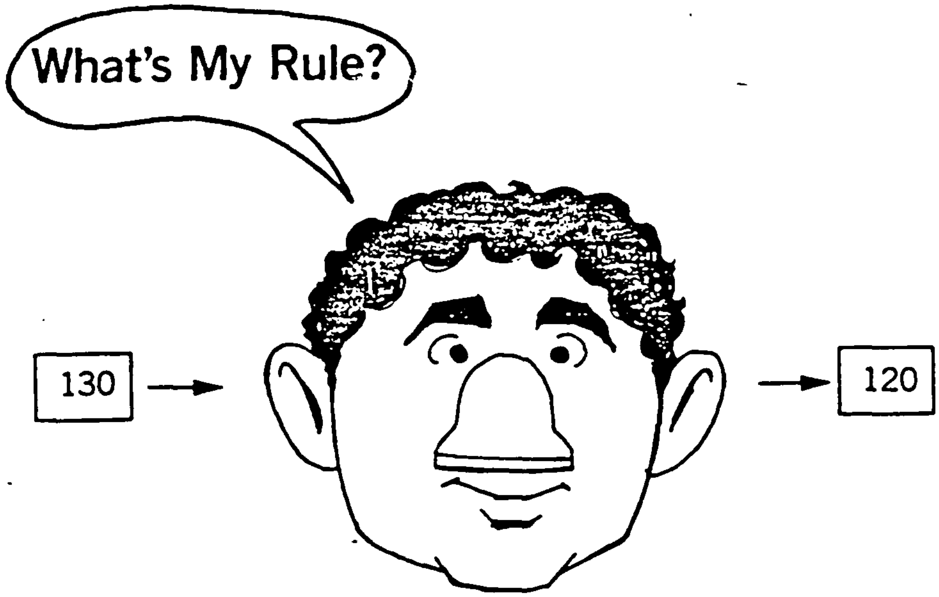
1. 46 →  → ☐

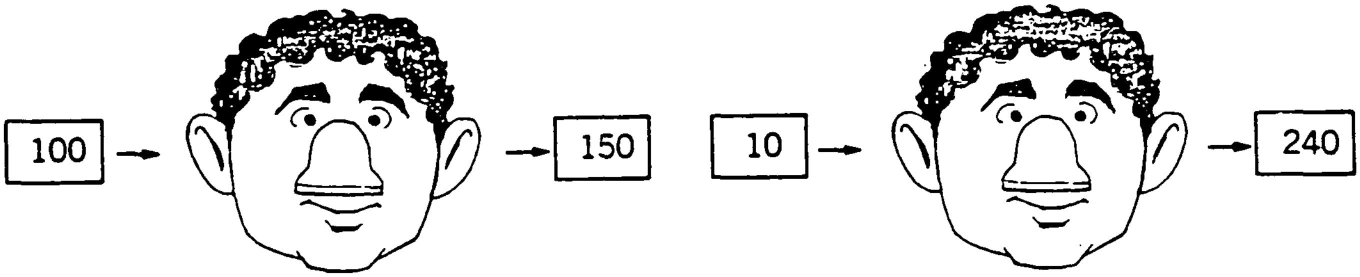
2. 63 →  → ☐

3. 100 →  → ☐

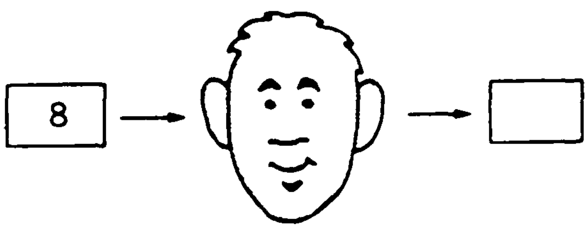
4. 188 →  → ☐

The answer to **4** is the speed in kilometers per hour of the fastest recorded pitch in baseball.

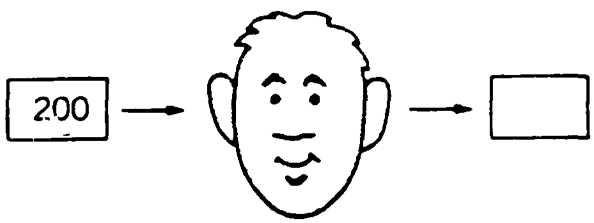




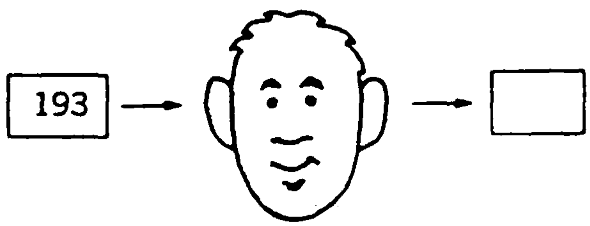
1.



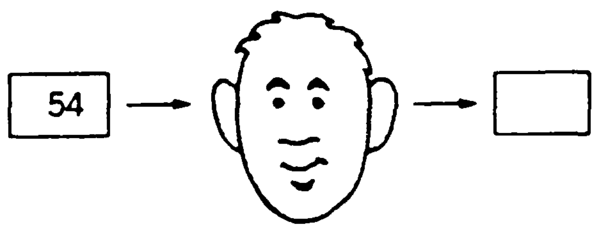
2.



3.



4.

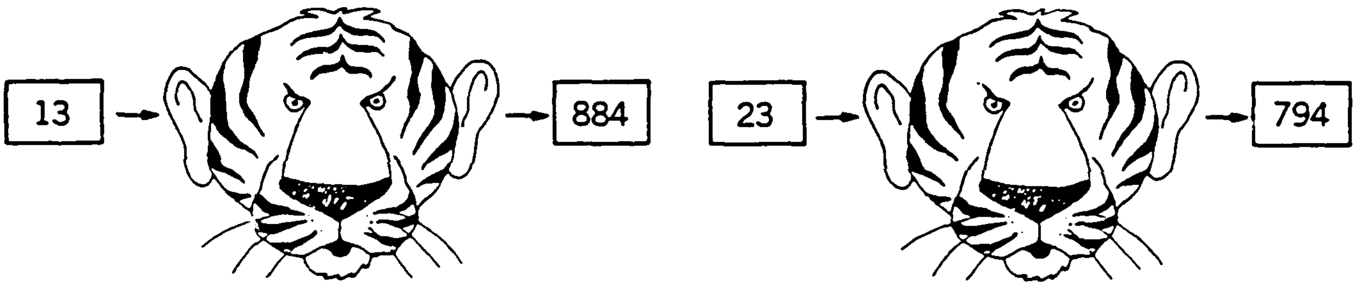


The answer to **4** is the weight in metric tons of the heaviest bell in the world.

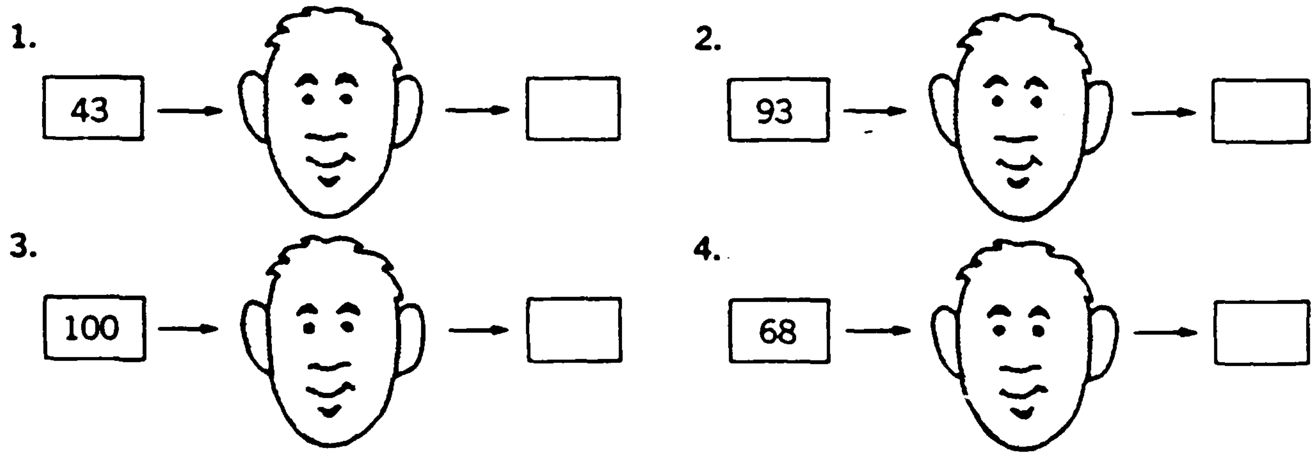
What's My Rule?

Activity 21

What's special about the "in" numbers here?

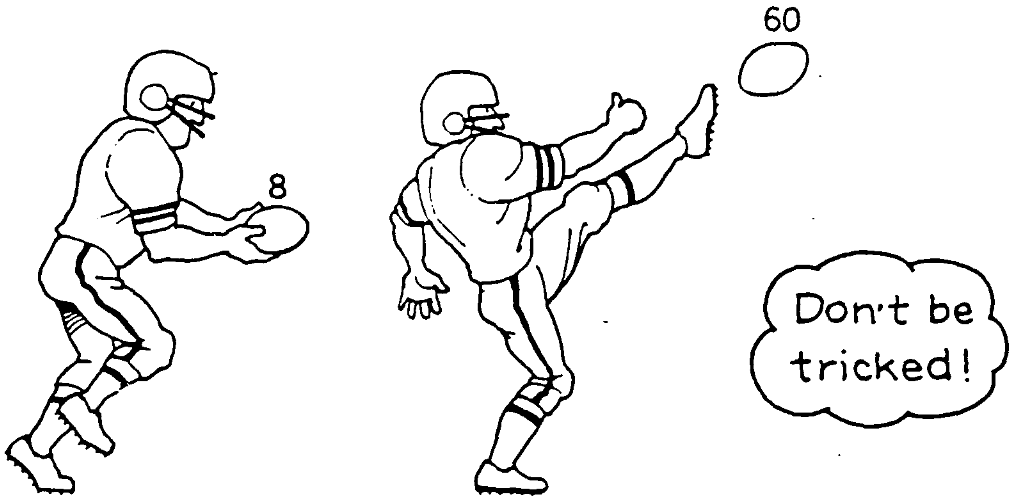


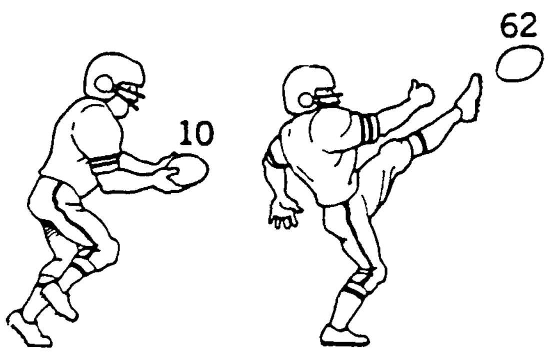
Try to guess what number comes out. If you need a hint, look up the answer for **1** on Answer List 1, for **2** on Answer List 2, for **3** on Answer List 3, and for **4** on Answer list 4.

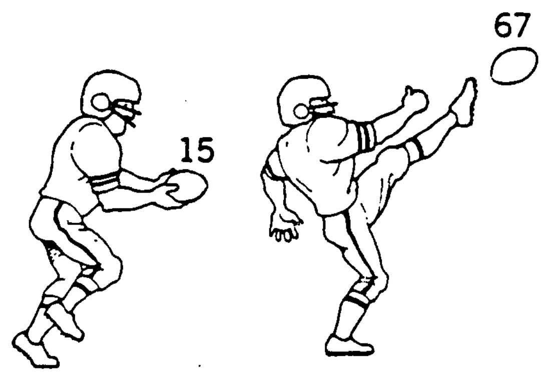


The answer to **4** is the weight in kilograms of the heaviest Indian tiger.

What Comes Out?







Try to guess what number comes out. If you need a hint, look up the answer for **1** on Answer List 1, for **2** on Answer List 2, for **3** on Answer List 3, and for **4** on Answer list 4.

1. In: 3

Out: ☐

2. In: 12

Out: ☐

3. In: 100

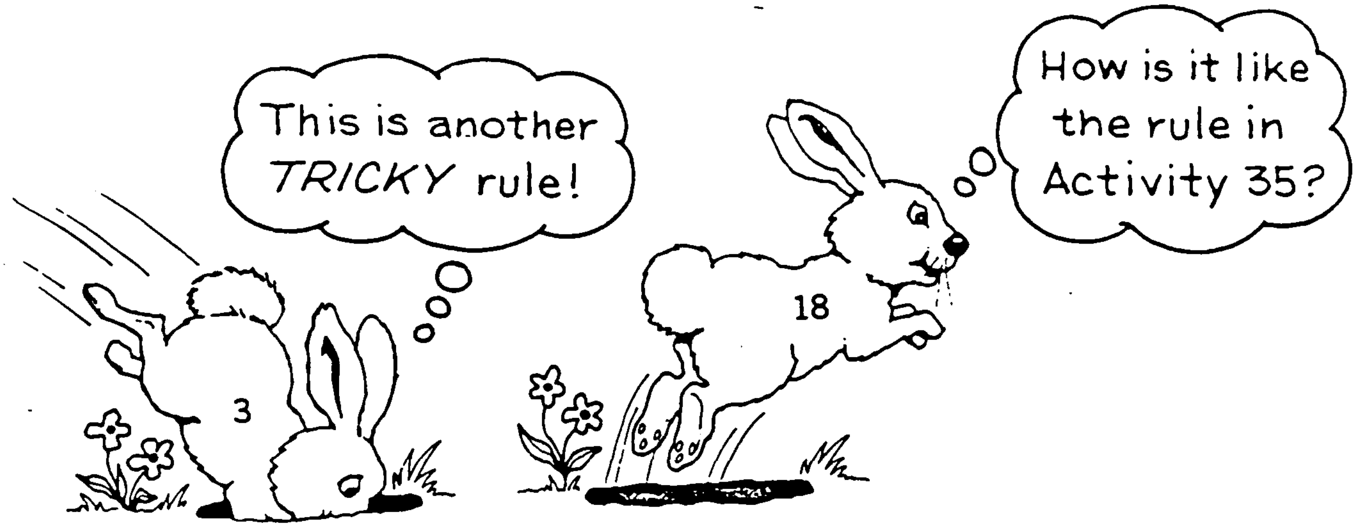
Out: ☐

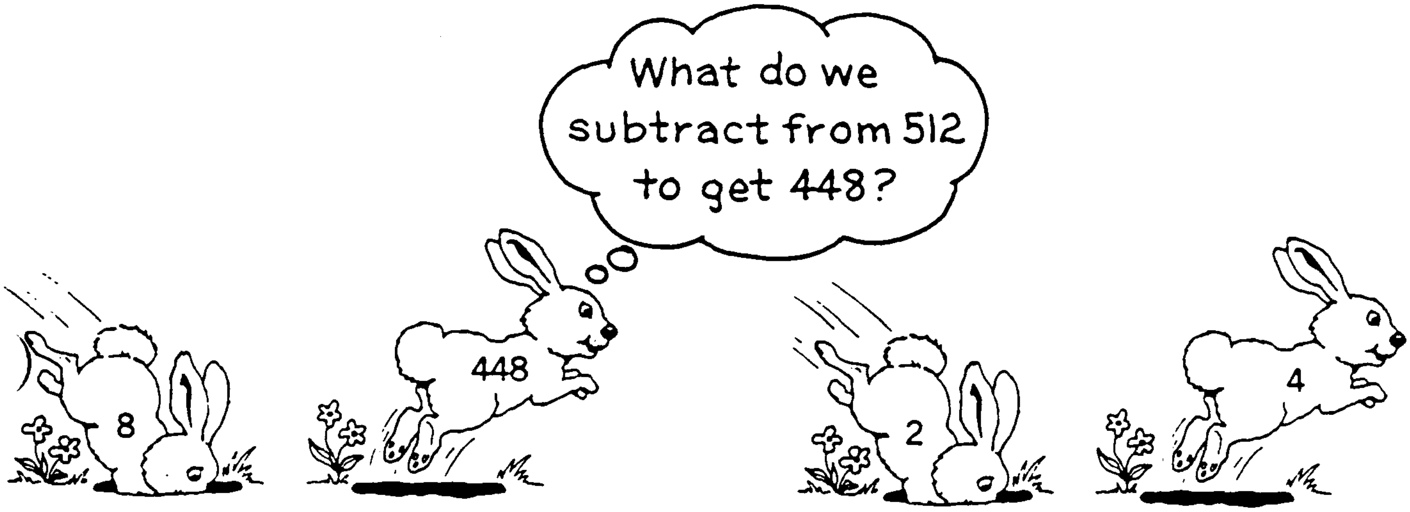
4. In: *n*

Out: ☐

The answer to **4** is an expression involving *n*.

What Comes Out?





Try to guess what number comes out. If you need a hint, look up the answer for **1** on Answer List 1, for **2** on Answer List 2, for **3** on Answer List 3, and for **4** on Answer list 4.

1. In: 4

Out:

2. In: 6

Out:

3. In: 1

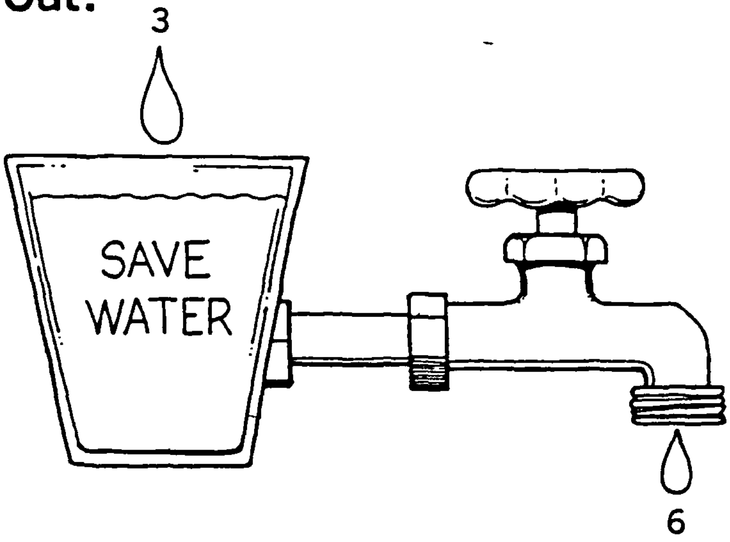
Out:

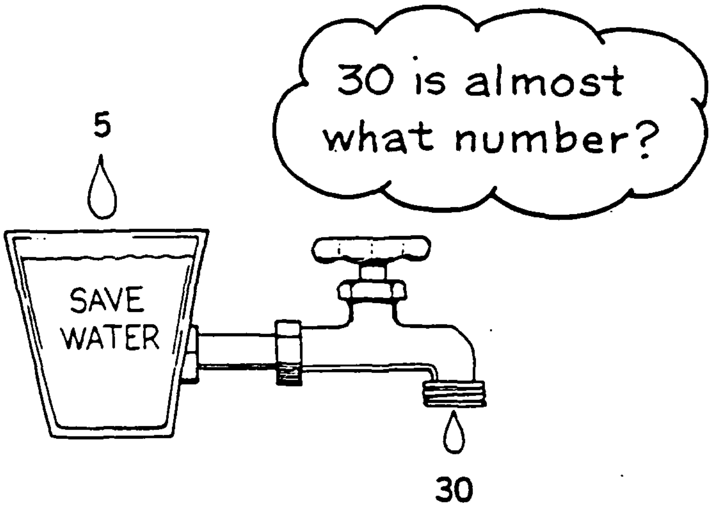
4. In: *n*

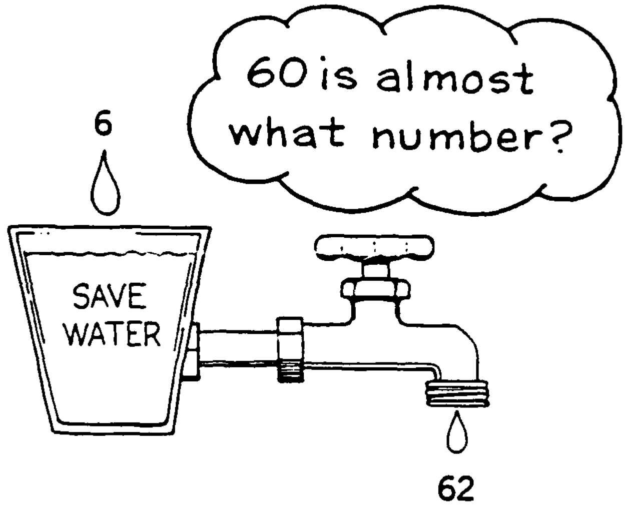
Out:

The answer to **4** is a slightly complicated expression involving *n*.

What Comes Out?







Try to guess what number comes out. If you need a hint, look up the answer for **1** on Answer List 1, for **2** on Answer List 2, for **3** on Answer List 3, and for **4** on Answer list 4.

1. In: 2

Out: ______

2. In: 1

Out: ______

3. In: 4

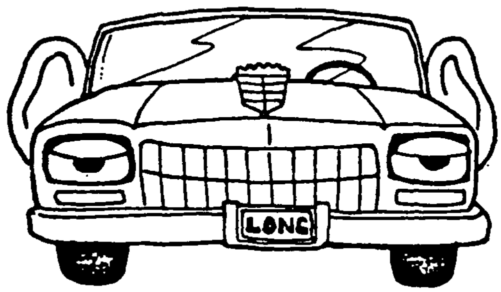
Out: ______

4. In: n

Out: ______

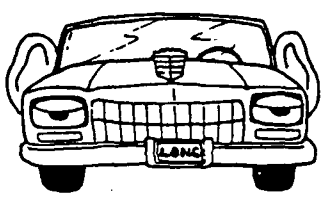
The answer to **4** is an expression involving n.

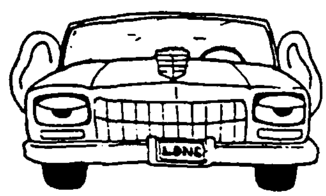
What's My Rule?

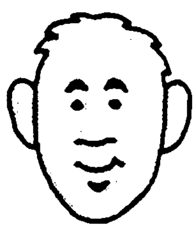
3 → 

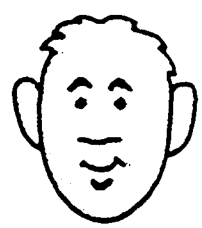
→ 39

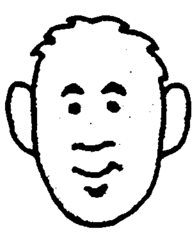
This may be very difficult.

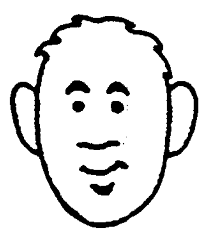
4 →  → 34

0 →  → 54

1. 6 →  → ☐

2. 8 →  → ☐

3. 10 →  → ☐

4. 9 →  → ☐

The answer to **4** is the length in meters of the longest car (a Cadillac).

PROBLEM SOLVING

1. At a party for mathematicians the host announces the following:

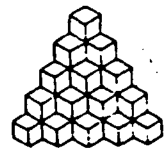
   I have three daughters. The product of their ages is 72. The sum of their ages is the same number as our house number. How old are my daughters?

   The guests confer, go outside to look at the house number, and then return to say that there is insufficient information to solve the problem. Thereupon the host adds this statement:

   My oldest daughter loves chocolate pudding.

   With this new bit of information the guests are able to determine the ages of the three daughters. What are these ages?

2. The tower to the right is made of 35 cubes in 5 layers. How many cubes are needed to form a similar tower with 10 layers?



3. What is my number?

   (a). It is a two-digit number.
   (b). It is a multiple of 6.
   (c). The sum of the digits is 9.
   (d). The ten's digit is one-half of the unit's digit.

4. The table to the right defines a binary operation on the set {a,b,c} if it is completed with elements of that set. How many binary operations can be defined on the set {a,b,c}?

+	a	b	c
a	__	__	__
b	__	__	__
c	__	__	__

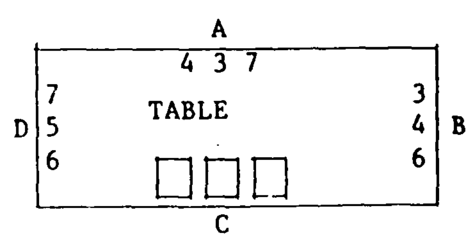
5. To play this game two decks of cards are needed: One deck contains the cards 3 through 7 of each suit of a bridge deck; the second deck has twelve question cards.

   To play this game, select groups of four players and place the two decks of cards in the middle of the group. After the decks have been shuffled, each player draws three cards, face down, from the bridge deck which he/she then (without seeing his numbers) props up in front of him/her for the other players to see. When play begins each player can see the number combinations of all the players except his own.

   The player who begins, draws the top card from the second deck, reads the question aloud for all players to hear and answers the question in accordance with the three combinations which he can see, before putting the card on the bottom of the deck.

   The following represents a record of play among players A, B, C and D with C's hand unknown to the reader. Answers of successive players are given as they occurred in actual play. The problem is to determine the implications of the answers; thus guessing the three numbers C had.

NO. OF PLAY	PLAYER	QUESTION	ANSWER
1	D	How many cards have numbers that are multiples of 2?	Three
2	A	How many 7's can you see?	Two
3	B	Do you see more odd or even numbers?	More odd
4	D	What is the sum of the numbers you can see?	Forty-two



Devising a Plan

a. Looking for a Pattern -

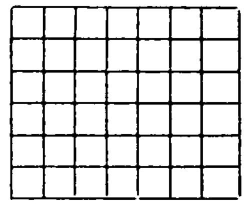
When the famous German mathematician Karl Gauss was a child, his teacher required the students to find the sum of the first 100 natural numbers. The teacher expected this problem to keep the class occupied for a considerable amount of time. Gauss gave the answer almost immediately. Can you?

b. Making a Table or Graph -

How many ways are there to make change for a quarter using only dimes, nickels, and pennies?

c. Using a Special or Simpler Case -

Using the existing lines in a square array of squares to form squares, how many different squares are there?



d. Identifying a Subgoal -

Kasey Kassion, a disk jockey for a 24-hour radio station, announces and plays each week' top forty rock songs on the radio all week. Suppose he decides to play the top song 40 times, the number two song 39 times, the number three song 38 times, and so on. If each song takes 4 minutes to play, how much time is left for other songs, commercials, news breaks, and other activities?

e. Using a Related Problem -

Find the sum of $1 + 4 + 7 + 10 + 13 + \ldots + 3004$.

f. Working Backwards -

Charles and Cynthia play a game called NIM. Each has a box of matchsticks. They take turns putting 1, 2, or 3 matchsticks in a common pile. The person who is able to add a number of matchsticks to the pile to make a total of 24 wins the game. What should be Charles' strategy to be sure he wins the game?

g. Writing Equations -

As he grew older, Abraham De Moivre, a mathematician who helped in the development of probability, discovered one day that he has begun to require 15 minutes more sleep each day. Based on the assumption that he required 8 hours of sleep on date A and that from date A he had begun to require an additional 15 minutes of sleep each day, he predicted when he would die. The predicted date of death was the day when he would require 24 hours of sleep. If this indeed happened, how many days did he live from date A?

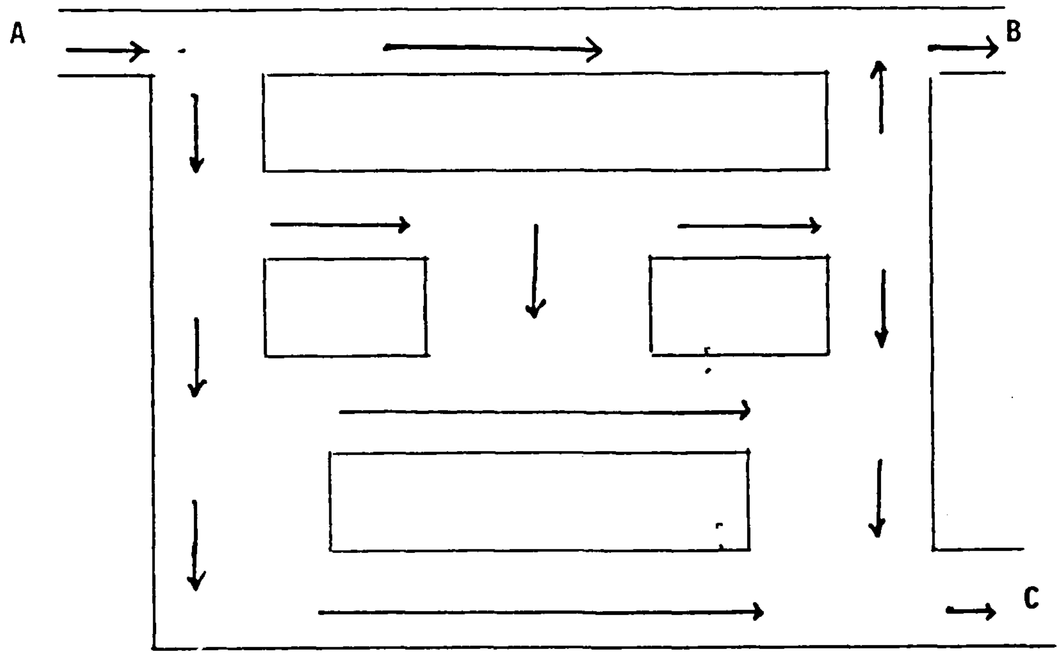
h. Using a Diagram or Model -

It is the first day of class for the course in mathematics for elementary school teachers, and there are 20 people present in the room. To become acquainted with one another, each person shakes hands just once with everyone else. How many handshakes take place?

i. Guessing and Checking -

Marques, a fourth grader, said to Mr. Treacher, "I'm thinking of a number less that or equal to 1000. Can you guess my number?" Mr. Treacher replied, "Not only can I guess your number, but I can guess it in no more than ten questions, provided that your answers to my questions are yes or no and are truthful." How could Mr. Treacher have been so positive about the maximum number of questions he would have to ask?

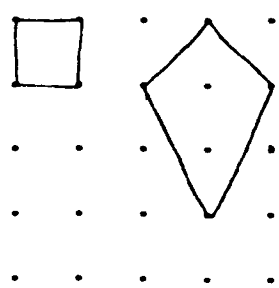
1. This diagram shows a network of one-way streets. At each corner where there is a choice, the traffic divides equally between the two possible directions. All cars leave the area at B or C. If 304 cars enter the area at A, how many of them exit at C?



2. Find the smallest number which when divided by eacn of the integers 2,3,4,5,6,7 and 8 will give in each case a remainder which is 1 (one) less than the divisor.

3. The gardener of a large estate had a panel of switches numbered 1 to 25, with each switch operating on of the 25 lights in the garden. One night the gardener flipped all of the switches, turning all 25 lights on. He decided that it was then too bright in the garden, so he flipped every second switch (2,4,6,8,...,24) again, turning off those lights. He began to enjoy this game, so he flipped switches again, this time, every third switch (3,6,9,...,24). He continued, flipping every fourth switch, then every fifth switch and so on all the way to "every " 25th switch. When he had finished, how many of the lights were on?

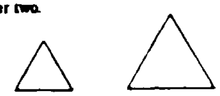
4. The area of the square is 1 square centimeter. What is the area of the Kite?



JULY

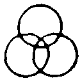
1

Given two equilateral triangles, find a third one whose area is the sum of the other two.



2

One circle divides a plane into two regions, two circles into four regions, three circles into ____ regions, four circles into ____ regions, and n circles into ____ regions.



3

1845b. **Georg Cantor**, founded set theory and contributed to analysis. Invented the Cantor set.

4

One square determines two regions, but two congruent squares can determine ten regions. Three squares determine ____ regions. Four congruent squares determine ____ regions. So n squares determine ____ regions.

5 **6**

The figures below contain one, three, and six rectangles, respectively.

How many rectangles are in [figure]? ____

How many rectangles are in [figure]? ____

How many rectangles are possible in a string of n squares? ____

7

What other numbers complete the pattern?

$5^2 - 5 = 4^2 + 4$

$7^2 - 7 = 6^2 + 6$

8 **9**

There are three rectangles in [figure].

There are nine in [figure] and eighteen in [figure].

How many are in [figure]? ____

If n is the number of stacks (columns) of rectangles that comprise each figure, then what is the total number of rectangles (of all sides) in each figure? ____

10

1748b. **John Playfair**, worked in Euclidean geometry, found substitute for Euclid's parallel postulate.

11

1780b. **August Leopold Crelle**, advocated learning mathematics as a way of developing rational thinking; discovered Abel and Jacobi.

12 **13**

There are six rectangles in [figure].

There are eighteen in [figure].

How many rectangles are in [figure]? ____

If n is the number of stacks (columns) of rectangles in each figure, then what is the total number of rectangles (of all sizes) in each figure? ____

14

1879b. **Albert Einstein**, stimulated interest in Riemannian geometry through his theory of relativity.

15

The Fibonacci sequence is 1, 1, 2, 3, 5, 8, ... Plot $1 \cdot x + 1 \cdot y = 2$ and $1 \cdot x + 2 \cdot y = 3$. Where do the resulting lines intersect? Pick three other consecutive elements in the sequence, such as a, b, and c in $ax + by = c$, and see if they go through the same point.

16

1750b. **Caroline Herschel**, astronomer, found three new nebulae and eight comets.

17

Two two-digit numbers can be formed with the digits 4 and 2, namely, 24 and 42. We know that $24 + 42 = 66$. Make two other two-digit numbers and add them together. What number divides all the sums?

18

1690b. **Christian Goldbach**, conjectured that all even numbers except 2 can be represented as the sum of two primes. What do you think?

$7 + 3 = 10$

$7 + 5 = 12$

$7 + 7 = 14$

...

19

Return to the problem for March 17. How does the sum of the two single digits compare to the sum of the two two-digit numbers? (For example, compare $4 + 2 = 6$ with $24 + 42 = 66$.)

20

How many triangles are in each shape?

$n = 1$ $n = 2$ $n = 3$ $n = 4$

How many more △ are in the figure for $n = 2$ compared to $n = 1$? For $n = 3$ compared to $n = 2$? What pattern can you find in the differences?

21

1884b. **George David Birkhoff**, first major American mathematician, contributed in many areas; ergodic theorem.

22

Find the decimal values to ten places for 2/7, 3/7, 4/7, 5/7, and 6/7. What patterns do you notice in these values?

$\frac{1}{7} = 0.14285714285\ldots$

$\frac{2}{7} = ?$

23

1882b. **Amalie Emmy Noether**, leader in the development of modern algebra.

24

1809b. **Joseph Liouville**, attempted to classify all algebraic functions; first proof of existence of transcendental numbers. A Liouville number is an irrational number x such that for any integer n, there is a rational number p/q such that $q > 1$ and $|x - p/q| < \frac{1}{q^n}$.

25

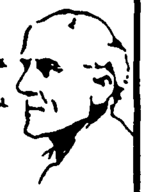
1798b. **Christof Gudermann**, worked with spherical geometry; saw value of using infinite series in calculus.

26

1773b. **Nathaniel Bowditch**, wrote about astronomy; worked as an actuary; his book *The American Practical Navigator* is still available.

27

1857b. **Karl Pearson**, founded field of statistics. Introduced chi-square test, Pearson's coefficient.



28

Continue this pattern for 4 more steps and then compare your result to $\sqrt{2}$. Try beginning with a fraction other than 3/2.

$\frac{3}{2} \to \frac{3 + 2(2)}{3 + 2} = \frac{7}{5} \to \frac{7 + 2(5)}{7 + 5} = \frac{17}{12} \to$

29

Find three consecutive integers that add to 75, three that add to 99, and three that add to 150. If n is a number that is the sum of three consecutive numbers, what divisors must it have?

30

1890b. **Stefan Banach**, one of the originators of functional analysis, developed theory of topological spaces. Banach algebra, Banach space.

31

1596b. **René du Perron Descartes**, laid foundation for analytic geometry.

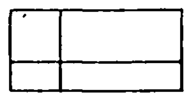
JULY

1 What percentage of the area of a circle is enclosed by an inscribed isosceles triangle one of whose sides is the diameter of the circle?

2 Dave computed an answer to be 12.5. However, in the last step of the computation he multiplied by 0.3 instead of dividing by 0.3. Assuming that Dave computed correctly, what is the correct answer?

3 About how many times will your heart beat during this month?

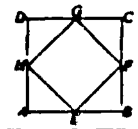
4 How many rectangles do you see in this figure?



5 What is the area of a square whose diagonal is one unit longer than the length of its side?

6 1634b. Pierre Herigone, first person to use the symbol ∠ for angle.

7 The area of square *ABCD* is 100 cm². *E* and *F* are the midpoints of sides on which they lie. What is the area of square *EFGH*?



8 Two more people are ahead of me in line than are behind me. There are three times as many people in line as there are people behind me. How many people are ahead of me in line?

9 1746b. Gaspard Monge, worked in descriptive geometry.

10 The first U.S. transcontinental railroad was completed on this date in the year ___?___.
The sum of the digits in the year is 24.
The ones and tens digits are multiples of 3.
The hundreds digit is one less than the ones digit.

11 1659. The division symbol, ÷, was first used by Johann Rahn.

12 Continue the pattern:

$1^3 = 1^2 - 0^2$
$2^3 = 3^2 - 1^2$
$3^3 = 6^2 - 3^2$
$4^3 =$ ______
$5^3 =$ ______

13 1753b. Lazare Nicolas Marguerite Carnot, politically active and mathematically concerned with the efficiency of machines.

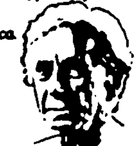
14 When the length of each side of a square is increased by 5 units, the area of the square is $2\frac{1}{4}$ times the area of the original square. What is the area of the original square?

15 What percentage of the states in the U.S. have names that begin with a vowel?

16 1718b. Maria Gaetana Agnesi, authored *Analytic Institutions*, a study of algebra, geometry, and calculus.

17 What is the karat rating of a gold ring that is 58 percent gold?

18 1872b. Bertrand Russell, philosopher, logician, coauthor of *Principia Mathematica*.

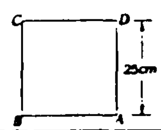


19 A "prime day" is a day such that both the *month* and the *day* are prime numbers. How many prime days occur in 1984? (Example: 2/2/84 is the first prime day.)

20 What is the value of a metric ton of nickels?

21 1858b. Edouard Goursat, contributor to analysis; "Cauchy-Goursat theorem."

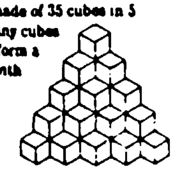
22 What are the dimensions of the square *EFGH* you must cut from the square *ABCD* so that the area of square *ABCD* is reduced by 25 percent?



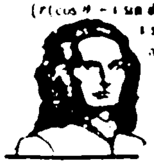
23 What three consecutive whole numbers have a product of 29 760?

24 Which pairs of two-digit consecutive numbers have squares that differ by a perfect square?

25 This tower is made of 35 cubes in 5 layers. How many cubes are needed to form a similar tower with 10 layers?



26 1667b. Abraham De Moivre, proved $(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$; formulated notion of the normal distribution curve.



27 Three straws are chosen from a set of nine straws whose lengths are 1 inch, 2 inches, 3 inches, ..., 9 inches. What is the probability that the three straws placed end to end, will form a triangle?

28 Choose any number, multiply by 3, add 5, multiply by 5, subtract 25, and divide by 10. Compare your result to the original number.

29 Choose any number, multiply by 3, add 9, add your original number, divide by 4, and subtract your original number. Compare your result to the original number. Write an equation using *n* for some number that describes the steps in this problem and in the problem for 28 May.

30 Try to write each integer from 1 through 32 as a sum of two or more consecutive positive integers. Does a pattern occur in the numbers that cannot be represented in this way?

31 Square the numbers between 10 and 30. How many palindromes do you get? (A palindrome reads the same forward as backward.) How many palindromes are between 20² and 30²? What would you expect for the squares between 30² and 40²? Check it out.

Calendar Problems (Set 3)

JULY

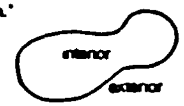
1
In every numeration base b is a three-digit integer that is $(b-1)$ times the sum of its digits. In base ten it is 198, since
$198 = (10+1)(1+9+8)$.
What numbers have this property for bases two through nine?

2
A fair coin with faces 0 and 1 is tossed repeatedly to see whether the ordered triple 111 or 011 turns up first. What is the probability that 111 will turn up first?

3
1777b. **Louis Poinsot**, established theory of regular star polygons.

4
Find three consecutive binomial coefficients in the ratio 1 to 2 to 3. That is,
$\binom{n}{k}:\binom{n}{k+1}:\binom{n}{k+2} = 1:2:3.$

5
1838b. **Camille Jordan**, worked in algebra and group theory; "Jordan curve theorem," "Jordan canonical form."



6
The right triangle with sides of length 5, 12, and 13 has the property that its area is equal to its perimeter. Find another right triangle with sides of integral length that has this property.

7
1871b. **Emile Borel**, contributed to analysis and probability.

8
1888b. **Richard Courant**, worked in physics, mini-max theory of eigenvalues; founded Courant Institute at New York University.

9
The first few factorials greater than 1 are clearly not perfect squares:
$2! = 2$; $3! = 6$; $4! = 24$; $5! = 120$.
Can any factorial greater than 1 be a perfect square?

10
1875b. **Issai Schur**, stated that every matrix is unitarily similar to a triangular matrix.

11
Guess by what rule the following equalities are composed, then make up another such equality of your own:
$12 \times 42 = 21 \times 24$
$13 \times 62 = 31 \times 26.$

12
For each odd positive integer n, show that
$1 + 9 + 9^2 + 9^3 + \cdots + 9^n$
is composite. For example,
$1 + 9 = 10 = 2 \times 5;$
$1 + 9 + 9^2 + 9^3 = 820 = 10 \times 82.$

13
1876b. **Erhard Schmidt**, worked on integral equations and Hilbert space theory.

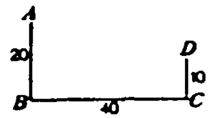
14
Gil is 17 years older than Sheila. If his age is written after hers, the result is a four-digit perfect square. The same statement could be made 13 years from now. Find Sheila's present age.

15
1850b. **Sofya Korvina-Krukovskaya** (Sonya Kovalevski), researcher in differential equations and algebra.

16
Find the sum of the series
$\frac{1}{2} + \left(\frac{1}{3}+\frac{2}{3}\right) + \left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right) + \left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right) + \cdots + \left(\frac{1}{100}+\frac{2}{100}+\frac{3}{100}+\cdots+\frac{99}{100}\right)$

17
At precisely what time between one and two o'clock is the minute hand exactly over the hour hand?

18
Timothy must get from point A to point D and must touch some point between B and C along the way. (For example, he could go from A to B to D; or A to C to D; or A to the midpoint between B and C and then to D.) What is the shortest length of such a journey satisfying these conditions?



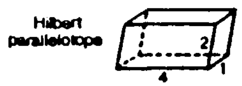
19

20
The ancient Egyptians had a rule that told them when, given two unit fractions with the second denominator twice the first, the sum of the two unit fractions would itself be a unit fraction.
For example, $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ is not a unit fraction, but $\frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$ is. What was this necessary and sufficient rule?

21

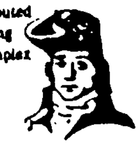
22
1874b. **Leonard Dickson**, known for his text on history of number theory.

23
1862b. **David Hilbert**, in 1900 posed 23 unsolved problems that stimulated mathematics research.



24
1798b. **Karl von Staudt**, worked in projective geometry.
1916 Browning, Montana, a record variation in one day, 44°F to −56°F. What is the record difference in degrees?

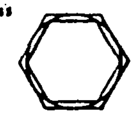
25
1736b. **Joseph Louis Lagrange**, astronomer, contributed new ideas for solving equations with complex variables.



26
1862b. **E. H. Moore**, developed general theory of limits.
1978 **Peter Dowdeswell** ate 13 raw eggs without shells in 2.2 seconds. How fast did he eat 1 egg?

27
1832b. **Charles Dodgson** (Lewis Carroll), logician, geometer, advanced voting theory; wrote about Alice.

28
1540b. **Ludolph van Ceulen**, computed π (Ludolph's number) to 35 places using Archimedes's method.



29
In bowling, a perfect score of 300 can be obtained in only one way: getting twelve strikes in a row. Can any other scores be obtained in only one way?

30
Friday the thirteenth has a reputation as being an unlucky day. Is it possible to have a year in which there are no Friday the thirteenths?

31
How many digits does the number $(9)^{(9^9)}$ have when written in base nine?

COMPUTER SCIENCE

The goals of the computer science course are to prepare students

- to be literate in the language and hardware of computer science,
- to develop algorithms using flowcharts and pseudo code,
- to understand more complex algorithm development using top down structured design, and
- to construct and debug programs in the BASIC language that employ control statements, string variables and arrrays, and that use data files, graphic techniques and subroutines.

COURSE OUTLINE

Unit I: Introduction/ Hardware Components

- Orientation and tour of campus computing facilities
- Terminology, history
- Introduction to BASIC
- Lab session: Terminal operation, complete questionaire at the terminal, run sample BASIC program

Unit II: Simple algorithm development and Implementation

- Flowcharts vs Top-down design
- Simple BASIC program
  INPUT; LET; PRINT (READ...DATA)
- Numeric and character string variables
- Lab session: Simple programs with INPUT, arithmetic computation and printout; programs with string variables

Unit III: Decision and Control Structures

- Use of pseudo Boolean variable
- IF...THEN; IF...THEN...ELSE
- Looping and iteration; FOR...NEXT; WHILE...NEXT
  Control by out of data; count control; flag or marker in data
- Accumulation

- Using external data files

- Lab session: Programs with decisions, string variables, word processing simulation; programs requiring counting and tallying

Unit IV: Top-down Structured Design and Problem Solving

- Subroutines (Procedures,Modules)

- GOSUB...

- Lab session: Programs with skills to date; develop a program to do a multiple choice quiz at the terminal

Unit V: Arrays and Tables

- Use of subscripted variables in single arrays

- Lab session: Assigned problems with array implementation; searching and sorting

Unit VI: Two Dimensional Arrays and Computer Graphics

- Two dimensional array concept as grid for graphics tablet or screen; consider points on screen or squares on grid

- Graphic highlights for two tone screen

- Color graphics - Slide presentation and terminal usage

- Lab session: Continue solution to more complex problems emphasizing arrays and tables; continue student projects

TEXT: Mandell, Steven L., Introduction to BASIC Programming, Second Edition, West Publishing Co., 1985.

DAILY ACTIVITIES

Lecture/discussion T,Th; Lab M,W,F
Activities, on a daily basis, are presented on the following pages.

WEEK 1: DAY 1   Lab - Begin Unit I

Tour of University computing facilities:

- Computer Center (bldg. 41); University- wide time-shared system; system presentation showing main frame, console, magnetic tapes, hard fixed and removable disks, and other equipment; this particular system is accessed through remote terminals during this course.
- Computing facilities listed below are in the mathematics and computer science areas in bldg. 32.
- College of Physical Science Computer Center ; comparasion of similar components as above.
- Computer terminal lab where lab work will be done during the course.
- Computer Engineering and Microprocessor Lab where microprocessors are shown at the component level; system boards, integrated circuits, floppy disks, and other components are shown and discussed.
- All tour activity is in small groups (half of each section) on a staggered basis that fits well with other first day orientation activities.

WEEK 1: DAY 2 Lecture/discussion

Continue Unit I; begin Unit II

- Computer Hardware; terminology

  Presentation of essential hardward diagrams relating file storage on disk and terminal network for communications. Note the analogy between the computer's central processing unit, memory, input/output devices and a human's brain, memory, senses for input and speech and writing for output.

- History

  Brief introduction to history of hardware and software.

- BASIC

  - Stored program concept
  - BASIC files
  - BASIC system commands: READY, LIST, NEW, OLD, SAVE, CATALOG, LOAD, RUN, DELETE line n
  - Simple program to INPUT, compute, PRINT
  - Numeric and string variables

ASSIGNMENT:

Read pages 2-11; pages 16-20 (skip sections on computers other than DEC system)

Worksheets Hardware Unit I, 1 and 2
History Unit I, 3 and 4 (due by week 4 day 1)

UNIT I - 3 WORKSHEET Complete and turn in Week 4: Day 1

Name ______________________ Date ____________

Identifying the Computer Hardware Chronology

INSTRUCTIONS: In the space provided below, list the significant events that occurred in the development of computer hardware during the time periods specified. Use the library, class notes and computer messages that you will receive. Be sure to include computer generations and the technology making the difference.

- 1950

1951 - 1957

1958 - 1964

1965 - 1969

1970 - 1975

1976 -

Name ______________________ Date ____________

Identifying the Computer Software Chronology

Instruction In the space provided below, list the significant events that occurred in the development of computer software during the time periods specified.

- 1950

1951 - 1957

1958 - 1964

1965 - 1969

1970 - 1975

1976 -

WEEK 1: DAY 3 Lab Unit I

Part I LOGIN as instructed in the lab
(review textbook page 10)
Remember your user ID is

0599.73001 for group 1

0599.73002 for group 2

The entire USERID is

0599.73001.AAAANNNN

where AAAA stands for the first 4 letters of your last name

NNNN stands for the last 4 digits of your social security number (or number given to you in class)

After LOGIN and you have the @ sign

- Read your mail (yes you have some)

  @RDMAIL

- DO Class Survey (Look over the survey on the following pages before you do the survey)

  @SURVY2

- LOGOUT

Part II Do Unit I Lab (on following pages)

Part III Play game if time permits

@PS:<GAMES> to see a list of the available games

@PS:<GAMES>gamename to play a game
NOTE: the game file is not normally available during lab or class time. It is available during free time.

Part IV Review accompanying lab notes:

- Review in your textbook from page 10; these are system commands to use the operating system. Other commands are available to you and shown in various system files.

- Reference Guide for BASIC (system commands) is on following pages for quick reference; it is also in your textbook.

```
THE PRECOLLEGE PROGRAM SURVEY
Written and developed by CLARK W SCOTT
YOU WILL BE ASKED QUESTIONS
AND ANSWERS ARE EXPECTED TO ALL QUESTIONS
THANK-YOU
what is your first name type it here.>

what is your middle name type it here.>

what is your last name type it here.>

what is your AGE

WHAT MONTH WERE YOU BORN

1.JAN  2.FEB  3.MAR  4.APR

5.MAY  6.JUN  7.JUL  8.AUG

9.SEPT 10.OCT 11.NOV 12.DEC

TYPE IN THE CORRECT THE NUMBER

WHAT DAY WERE YOU BORN

WHAT YEAR WERE YOU BORN <TYPE NUMBERS>
EXAMPLE: IF YOU WERE BORN IN 1971 TYPE 71
EXAMPLE: IF YOU WERE BORN IN 1970 TYPE 70

what is your ADDERESS

WHAT DO YOU EXPECT TO DO AFTER HIGH SCHOOL
type 1 or 2 or 3
1.college   2. work    3.other

WHAT ARE YOUR CAREER PLANS RIGHT NOW
type the number of your choice
        1.math  2.engineering  3.science  4.teaching
        5.medicine  6.lawyer  7.computer science  8.other

what types of activities or hobbies do you enjoy / are
involved with in your spare time
example camping,football,bowling,potery,reading,shopping
type in your top three
hobby or activity 1>:

hobby or activity 2>:

hobby or activity 3>:

DO YOU HAVE ACCESS TO A  PERSONAL COMPUTER OUTSIDE OF SCHOOL OR OUTSIDE
OF THE PROGRAM YOU ARE NOW IN
YES OR NO
```

```
Indicate THE LEVEL YOU FEEL MOST COMFORTABLE IN COMP SCI
type 1, 2, 3, 4, 5, or 6
        1. simple statements
        2. Decision and loops
        3. single arrays
        4. tabels
        5. two dimension arrays
        6. hi-res graphs

IS THIS INFORMATION CORRECT
OR IS THERE SOMETHING THAT NEEDS TO BE CHANGED
TYPE YES OR NO
1.FIRST INTI:
2.LAST NAME:
3.MIDDLE INTI:
4.AGE:0
5.MONTH OF BIRTH:0
6.DAY OF BIRTH:0
7.YEAR OF BIRTH:0
8.ADDERESS:
9.AFTER HIGH SCHOOL PLANS:
10.HOBBIES OR ACTIVITIES
   A.             B.             C.
11.PERSONAL COMPUTER:
12.CAREER PLANS:0
13.YOUR LEVEL:0

type the number of the wrong section
```

Unit I Lab

Objective: to become familiar with the terminals for use with BASIC PROGRAM.

to be able to enter and run a simple program

STEP 1: At home, review these notes and the attached reference guide.

STEP 2: At the lab - observe the terminal keyboard, be sure you can locate:

- -the carriage return
- -the alphabet
- -digits
- -* + - /
- -, comma, parenthesis (), single quote mark ', double quote mark "
- - control key
- -escape key
- -tab key
- -rub out or delete key
- -shift key

STEP 3: Login as instructed at the lab. If the phone connection is already made, only press **control C** to login. Your ID is 0599.73002 . AAAANNNN, First 4 letters of your last name
Digits according to your class list PASSWORD NOODLES

STEP 4: Type in and run sample program

Ⓔ is the escape key

- use **rub out** or **delete** ← key to correct a character just typed.

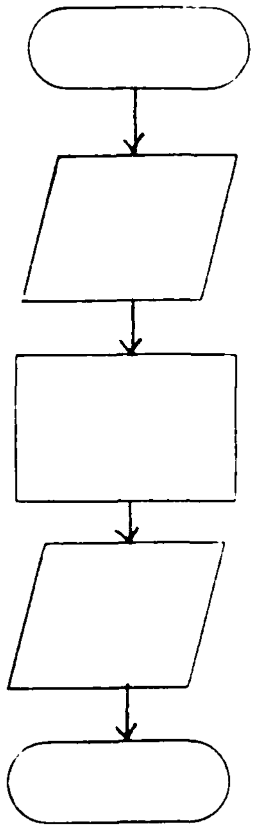
- to correct an entire line, just retype the line number and retype the correct statement.

^C SAMPLE PROGRAM - You type lower case; MACHINE TYPES UPPER CASE

```
@basic

READY
new unitlab

READY
00010 rem this is a remark statement
00020 rem type your name here
00030 read a,b,c
00050 data 3.,4.,8.
00080 let n = 2
00090 let s = a + b
00100 let d = a - B
00110 let p = a * b
00120 let q = c/b
00130 let e = a**n
00140 print 'sum=';s,'diff=';d, 'prod=';p
00150 print 'quotient';q, 'power';e
00160 end

save

READY
run
sum= 7        diff=-1        prod= 12
quotient 2    power 9

READY
bye
```

WEEK 1: DAY 4 Lecture/discussion Continue Unit II

Flow charts for algorithm development



	BASIC STATEMENTS
Start	REM
Get some data	INPUT
Do something with it	LET
Show your results	PRINT
Stop	END

BASIC programming

REM, variables vs constants, INPUT, LET, PRINT

Review stored program concept by examples of program traces

ASSIGNMENT: Read and study (pay special attention to program examples) page 21-42

Do worksheet Unit II, 1

Prepare lab programs for Week 2 day 1

WEEK 1: DAY 5 Lab ---Holiday

WORKSHEET Unit II-1 Variables, Stored Program Concept, Simple Program

1. Mark the following as valid or invalid variable names.

A	________	A5	________
B$	________	1B	________
NUMBER	________	NAME$	________
$YES	________	OK	________

2. Choose meaningful variable names for the following values.

Your year of birth	________	Your Grade average	________
Your name	________	Your favorite song	________

3. What is the value of SUM after the following program segment is executed?

```
A = 2
B = 8
c = B/A
SUM = A  + C
```

____

4. Trace the values of the variables in the following program. Use a ? if the values are unknown.

a.

	A	B	C
A = 12	____	____	____
n = 4	____	____	____
B = A * N	____	____	____
C = A/N	____	____	____
A = C	____	____	____
B = A + C	____	____	____

b.

	X	Y	Z
INPUT X,Y	____	____	____
Z = X + Y	____	____	____
X = Y + X * Z	____	____	____
GREET$ = "HELLO"	____	____	____
Y = Z * X	____	____	____

Values used at INPUT statement are 4,3

5. Do debugging exercises Page 39 1 & 2.

WEEK 2: DAY 1 Lab Unit II Simple Programs

Objective: to write and run simple BASIC programs

Write programs in the BASIC language for of the following problems. Be sure to include REM statements at the beginning to show the problem title, problem description, and your name. Remember the REM (remark) is the way to document your program; this statement may appear at any line in the program.
Run your programs on the computer in lab.
Note: Read pages 47-50 on printing output.

1. The Dairy Delight ice cream store needs a program which will calculate the amount of ice cream to order for milkshakes. Write a program which requests the user to enter the number of milkshakes he or she wishes to make. Each milkshake requires 6 ounces of ice cream. Your output should look like this:

   FOR XXX MILKSHAKES YOU WILL NEED XXX OUNCES OF ICE CREAM

2. In professional football, a touchdown counts 6 points, an extra point after a touchdown counts 1 point, a safety counts 2 points, and a field goal counts 3 points. Find the total score (TOTAL) for a team, if the number of touchdowns (DOWNS), safeties (SAFE), extra points (EXTRA), and field goals (FIELD) are entered as input. Remember to provide prompt messages for proper input and to provide clear print labels for output.

WORKSHEET Unit II- 2

Evaluate the value of the following expressions used in assignment (LET) statements.

Values: A = 2 B = 6 C = 1 D = 8 E = 5

```
LET X = A*B+C

LET X = A+ B / A

LET X = A * (E + C)

LET X = D + A ** 3

LET X = (A + B) * (C-D)

LET X = B ** 2 / (C+E)

LET X = (A + B) * (D - E)

LET X = A + B * C - D **2
```

WEEK 2: DAY 2 Lecture/discussion Complete Unit II; begin Unit III

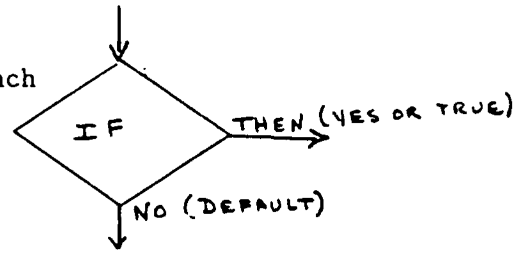
Topics

- Hierarchy of operations in assignment statement
  parentheses
  ** exponentiation
  * multiplication or / division
  + addition or - subtraction

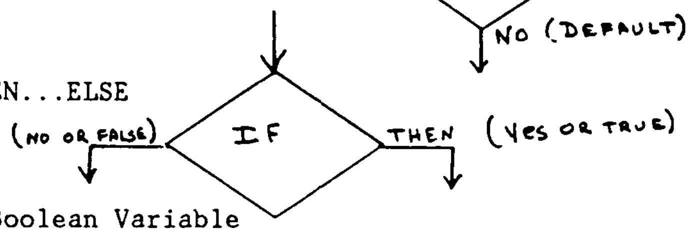
- Decision and control

  GOTO as an unconditional branch

  IF...THEN



  IF...THEN...ELSE



- Pseudo Boolean Variable

  Use of a flag variable to express "true" or "false"; current usage in BASIC allows:

  FLAG value of 0, FLAG is false
  FLAG value of 1, FLAG is true

- Loops with IF ...Then...GOTO

  Counting, summing and accumulation

ASSIGNMENT: Read pages 60-64; pages 67-71
Do worksheets Unit II-2 and Unit III-1
Begin lab programs for Week 2: Day 3

WORKSHEET Unit III-1 Decision and Control - IF Statements

1. Show what is printed as a result of the following:

   IF A > B THEN PRINT "A GREAT"

   a. A is 12   B is 6

   b. A is 6   B is 12

   c. A is 6   B is 6

2. Write a program segment that prints "EXCELLENT" for a grade greater than 90.

3. Write a program segment that assigns to the variable LEAST, the smallest value contained in the variables X and Y.

4. Write a program segment that prints "NEEDS HELP" for a salesman whose sales (variable SALES) is less than $20,000; and prints "DOING OK" when SALES is greater than $20,000.

WEEK 2: DAY 3   Lab Unit III - Decisions and Control

Write and run programs for two of the following problems.

1. Objective: Use of String Variables
   Decision making IF ... THEN
   Appreciation of computer word processing

Write a program to produce a form letter (or other form text such as a greeting card). The program must have at least two variable values to input (such as name , grade, age, or occasion) and at least one decision to provide alternate print messages (such as "son" or "daughter"). See sample program packet in your file.

2. Objective: Decision making in programs
   Appreciation of programs for interactive computer aided instruction

Design a four question, multiple-choice quiz for one of your classmates to take at the terminal. The quiz questions will be "asked" by the terminal and the answer will be received by the terminal. The quiz will also be graded by the program. The quiz may be about material that you have studied in this class (what about from your history sheets?) or about other material. Try to think about your own experiences with computer aided instruction. Be sensitive to interactive responses. The computer should congratulate a correct answer and provide help for an incorrect answer.

3. Objective: Counting and Summing in Loops

Write a program to poll your classmates for their opinion about a particular subject or candidate for office. You could ask about favorite candidates in the last presidential election (or favorite song, movie, or class).
Example: The election - The responses are coded into numbers as follows: 1 Jimmy, 2 Ronny, 3 Teddy, 4 George

Each person gets to enter a number for his or her choice. The program should print as shown below.

Name	Number	Percent
Jimmy	4	25
Teddy	4	25
George	4	25
Ronny	4	25

WEEK 2: DAY 4   Lecture/discussion   Unit III - Loops

Objective: to understand and to be able to use control structures for looping

Review loops used before with IF and GOTO. Discuss pitfalls of the GOTO and methods of avoiding "spaghetti" code. Lead to value of the looping control structures to produce more error free code.

```
FOR                for loop control by counting
.
.
.
NEXT
```

This statement is in standard BASIC and is available on all systems. Note that 1) the stepsize controls the increment or decrement in the loop; 2) a program may exit the loop by an IF ... THEN... upon encountering a particular condition (but here again we have a GOTO situation that could have been avoided by a WHILE...NEXT controlled loop). Refer to text for details of the FOR...NEXT.

Review notion of a "pseudo" Boolean variable or flag. This kind of variable is very useful when using the statement:

```
WHILE   condition
.
.
.
NEXT      (in textbook see shadow box page 86; NEXT is WEND)
```

We use this WHILE statement for loop control when we wish to exit the loop (based on a condition that is observed and marked by a flag during the loop) prior to observing all values. The flag that is used here is similar to a race car driver who continues to drive around the track until observation of a checkered flag to mark the end lap.

```
FLAG = 1
WHILE FLAG
  INPUT VALUE
  IF VALUE < 0 THEN FLAG = 0
  .
  .    DO SOMETHING WITH VALUE
  .
NEXT
```

exit loop when FLAG = 0 or false

OR

```
   INPUT VALUE
-> WHILE VALUE < 0    ---------+
     PRINT VALUE               |
     INPUT VALUE               |
-- NEXT                        v
                      exit loop when value  < 0
```

Summary: Use FOR when expecting to observe all values in loop up to the counting limit. Use WHILE when there is a need to exit due to condition other than counting or before the counting limit is reached.

Remember four factors used to control loops:

1. Set initial values or condition
2. Observe condition; exit or set exit flag if condition is met.
3. If exit condition not met; adjust test value by INPUT again or some other computed change such as increasing the count.
4. Mark end of loop for branch point.

ASSIGNMENT: Read and study pages 77-89

Do worksheet Unit III-2

Begin programs for lab Week 2: Day 5 (complete any previous labs first)

1. Which of the following FOR/NEXT loops is incorrect?

a.
```
10 FOR I = 20 TO 1 STEP -10
20   READ A
30 NEXT I
```

b.
```
10 FOR N = 10 TO 5 STEP -1
20   READ X,Y,Z
30   LET A = (X+Y+Z)/3
40   IF A > 100 THEN 60
50   PRINT A
60 NEXT N
```

c.
```
10 FOR J = 1 TO 2
20 FOR I = 1 TO 3
30 FOR N = 1 TO 4
40   PRINT J,I,N
50 NEXT N
60 NEXT I
70 NEXT J
```

d.
```
10 FOR J = 1 TO 5
20   READ X
30   GO TO 10
40 NEXT J
```

2. What output will be printed for the following program:

```
10 FOR I = 1 TO 5
20   PRINT I,
30 NEXT I
40 END
```

3. What output will be printed from the following program:

```
10 FOR K1 = 3.7 TO 4.1 STEP .1
20   PRINT K1,
30 NEXT K1
40 END
```

4. What output will be printed from the following program:

```
10 FOR A = 10 TO 4 STEP - 2
20   PRINT A,
30 NEXT A
40 END
```

5. What output will be printed from the following program:

```
10 LET S = 0
20 FOR E = 1 TO 5
30   LET S = S+E
40 NEXT E
50 PRINT E,S
60 END
```

6. What output will be printed from the following program:

```
10 LET A = 2
20 FOR B = A to 3*A
30   LET C = B
40   PRINT C,
50 NEXT B
60 END
```

7. Examine the following program:

```
10 LET N = 1
20   FOR I = 1 TO 3
30   LET N = N*I
40   PRINT N,
50 NEXT I
60 END
```

a. What will be printed?

b. What will be printed if line 20 is replaced with 20 FOR I = 1 3 STEP 2

c. What will occur if line 20 is replaced with 20 FOR I = 3 TO 1

d. What will be printed if line 20 is replaced with 20 FOR I = 3 TO 1 STEP -1

e. What will occur if the following line is added to the original program: 35 IF N >= 2 THEN 50

8. Examine the following program:

```
10 LET S = 0
20 INPUT M
30 FOR I = 1 TO 3
40   LET S = S+M
50 NEXT I
60 PRINT S
70 END
```

a. How many times will the computer stop and ask for a number to be typed in?

b. If the number 5 is typed in when line 20 is executed, what number will be printed at line 60?

c. If the line number of the INPUT statement is changed from 20 to 35, how many numbers will have to be typed in during the execution of the program?

d. If the change suggested in c above is made and the values 5, 6, and 7 are typed in, what number will be printed?

WEEK 2: DAY 5 Lab Continue Unit III Decisions and Loops

1. Write a program to produce the following

```
*
**
***
****
*****
******
*******
```

2. What about ? (optional problem)

```
      *
     ***
    *****
   *******
  *********
 ***********
*************
 ***********
  *********
   *******
    *****
     ***
      *
```

3. Objective: Appreciation of programs for game playing

Write a flowchart, logic diagram, or pseudo code for your algorithm. Then write the program for the game HILO. In this game, the computer selects a number from 1 to 10. The player tries to guess this number in 5 tries or less. If the guess is too high, the computer responds HI. If the guess is too low, the computer responds with LO. The program should let you play again if you wish.

NOTE: The computer selects the number from 1 to 10 by using the random number generator. N = INT (10 * RND +1)

where N is the number; INT is the integer function; RND is the random number generator. Refer to class discussion for functions. Insert the statement RANDOMIZE at the beginning of your program only after you are fairly certain it is correct.

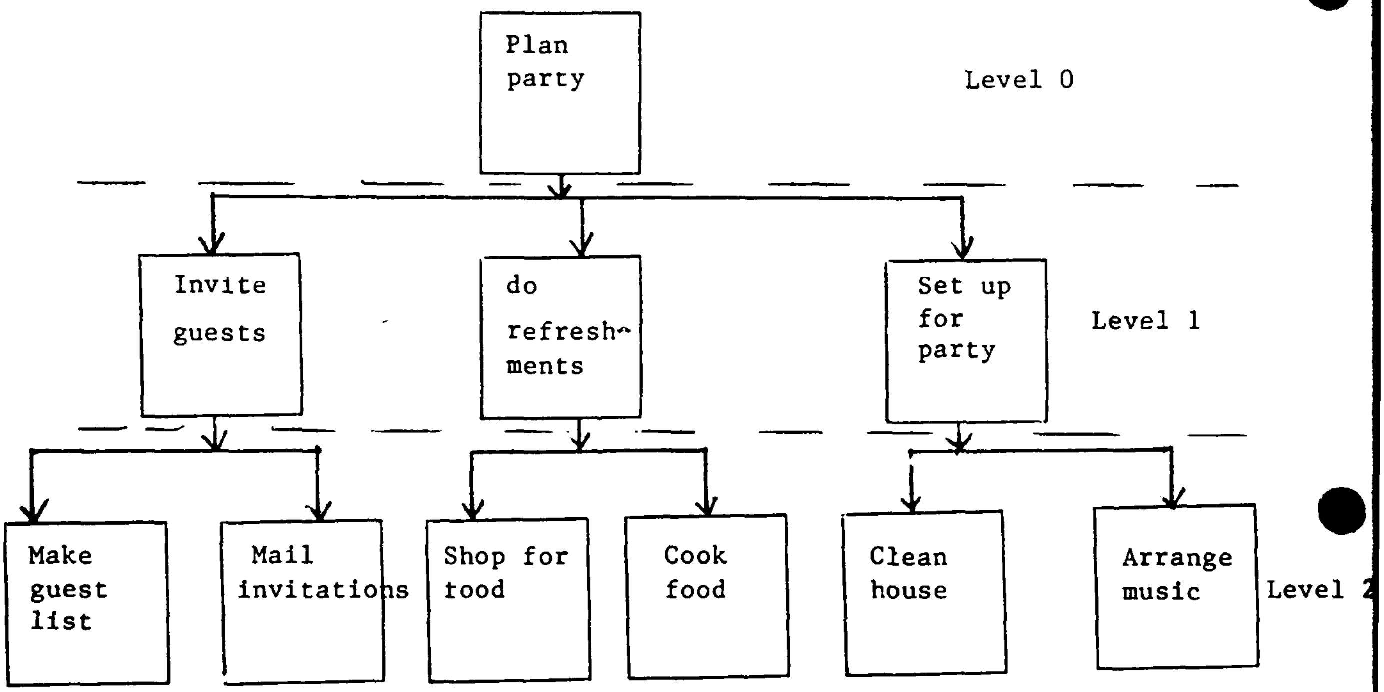
WEEK 3: DAY 1 Lab Unit III Decision and Control

Continue programs assigned to date

WEEK 3: Day 2 Lecture Unit IV Top-down Design

Problems are broken into sub-problems (called procedures, subroutines, or functions). The top-down design diagram looks like the executive management chart for a large company. Each management task relates to its own level and "passes the buck" the detail down to the next level to worry about. The top-down design is drawn in levels, beginning with the main module which flows to a level that does get data; do something with it; display your answers. Each of these levels is in turn broken down to push the detail down to a lower level.

The following diagram shows the design process applied to something like the task of planning a party.



GOSUB - the subroutine statement to implement some form of structure to BASIC programs

ASSIGNMENT: Read pages 104-107
Do worksheet Unit IV-1

WORKSHEET UNIT IV - 1   SUBROUTINES   GOSUB

1. What output is generated from the following program?

```
10 REM PAYROLL PROGRAM
20 REM R IS RATE, H IS HOURS
30 READ R,H
40 GO SUB 70
50 PRINT "GROSS PAY IS",G
60 STOP
70 REM GROSS PAY SUBROUTINE
80 LET G = H*R
90 RETURN
100 DATA 7,40
110 END
```

2. What output is generated from the following program?

```
10 PRINT "X","X↑2","SQR(X)"
20 LET X = 1
30 GO SUB 90
40 LET X = 4
50 GO SUB 90
60 LET X = 36
70 GO SUB 90
80 STOP
90 REM SUBROUTINE
100 PRINT X,X^2,SQR(X)
110 RETURN
120 END
```

3. What output is generated from the following program?

```
10 READ S
20 DATA 400,1200
30 GO SUB 60
40 PRINT C
50 GO TO 10
60 REM SUBROUTINE
70 REM OUTPUT VARIABLE IS S
80 REM OUTPUT VARIABLE IS C
90 LET C = .10*S
100 IF S > 100 THEN 120
110 RETURN
120 LET C = C+100
130 RETURN
140 END
```

WEEK 3: DAY 3   Lab   Unit IV   Top-down Design; Subroutines

Do one of the following programs.

1. Easter Sunday - The data for any Easter Sunday can be computed as follows: Let X be the year for which it is desired to compute Easter Sunday. All variables are integers. Use the integer function shown at the bottom of the page to force the variables to be integers after computation.

Let A be the remainder of the division of X by 19.
Let B be the remainder of the division of X by 4.
Let C be the remainder of the division of X by 7.
Let D be the remainder of the division of (19A + 24) by 30.
Let E be the remainder of the division of (2B +4C + 6D + 5) by 7.

The date for Easter Sunday is March (22 + D + E). (Note that this can give a date in April. Make the adjustment). Write a program that prints out a table showing the dates of Easter Sunday from 1965 to 1980.

2. Coins - Write a program to find the minimum number of rolls of coins (quarters, dimes, nickels, pennies) in a given amount of money. Also calculate the minimum number of coins in any amount left over. Use only integer arithmetic in solving this problem. Note the integer function and remaindering function at the bottom of the page.

Input the total amount of money $66.69
Output: Print a line for each coin, giving the number of rolls of that coin. If there are none, print an appropriate message. Be sure to label all results.

Print the amount left over in cents and then the minimum number of coins to make up that amount.

Note: The amount in dollars for a roll of each coin is

quarters	$10.00
dimes	5.00
nickels	2.00
pennies	.50

--------------------Notes on needed Functions-----------------------

Remaindering or Modulo function

MOD%(A,B) returns the integer result of A mod B, which is the remainder of A/B.

To force integer truncated division

Result = INT(A/B)

WEEK 3: DAY 4 Unit V Arrays and Tables

The following topics are discussed for one-dimensional (single) arrays or tables.

DIM dimension; to save space for the array

Subscripts and pointers

Use of FOR...NEXT with arrays

Various operations on arrays

Compute average; find largest element; parallel arrays

ASSIGNMENT: Read pages 124-130
Do worksheet Unit V-1
Begin lab Week 3: Day 5

WEEK 3: DAY 5 Lab Unit V Arrays and Tables

Objective: Reading stored data files from disk
Understanding and using one-dimensional arrays

Review class notes on reading files; run these statements to read and see the file.

```
OPEN 'CLASS' FOR INPUT AS 1
FOR I = 1 TO 40
  INPUT #1, LAST$, FIRST$, GRADE, CAREER, HOBBY$
  PRINT LAST$, FIRST$, GRADE, CAREER, HOBBY$
NEXT I
CLOSE #1
END
```

Use the above example and your notes for the problems. You may read the information into parallel arrays, however this is not required.

1. Find your name in the file and print out your hobbies.

2. Find a friend (or friends) in our group with the same career plans.

3. Find a friend (or friends) in our group who is in the same grade as you are and who has one of the same hobbies as you do.

Having a problem?? Why is #3 more difficult? What could have

helped? How should the survey have asked the question about hobbies?

WORKSHEET UNIT V - 1 ARRAYS AND TABLES

1. Assuming an appropriate DIM statement has been executed, which of the following array elements are valid? (Yes or No)

____ a. X(1)
____ b. Z(14)
____ c. X(I)
____ d. Z1(5)
____ e. (X)
____ f. K(Z*17-Y)
____ g. Z(X(I))
____ h. X(1,2)
____ i. Y(I,J)
____ j. Z(14,6)
____ k. (Z7)
____ l. Y(X(I),X(J))
____ m. B(X+1,X1)
____ n. S1(X,Y)

Use the following program to answer questions

```
10 DIM
20 DIM A(6,3)
30 FOR I = 12 TO 1 STEP -1
40   READ X(I)
50 NEXT I
60 FOR J = 1 TO 3
70 FOR K = 6 TO 1 STEP -1
80   READ A(J,K)
90 NEXT K
100 NEXT J
110 DATA 1,2,3,4,5,6,7,8,9,10
120 DATA 20,30,40,50,60,70,80,90
130 DATA 14,16,21,24,27,32,81
140 DATA 15,40,41,42,43
150 END
```

2. The value stored in X(4) will be

a. 4
b. 9
c. 8
d. 3

3. The value stored in A(2,3) will be

a. 42
b. 21
c. 41
d. 24

4. Examine the following program:

```
10 DIM C(25)
20 READ N
30 DATA 5,21,10,13,7,6
40 FOR K = 1 TO N
50   READ C(K)
60 NEXT K
70 END
```

a. Will the program run if line 10 is deleted entirely?

b. What value will be stored in array element C(3) before the loop is started?

c. What value will be used in array element C(3) after the loop is done?

WEEK 4: DAY 1   Lab   Begin Projects

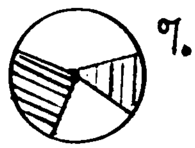
Complete any assigned programs to date. Suggested projects are shown below. Projects may be completed with groups of up to four students.

1. Enhance the multiple choice quiz or opinion poll program to include screen graphics to make the interactive program visually more readable and more attractive.

2. Enhance opinion poll to output a bar graph showing % of vote for each candidate (or each choice). The bar graph may be represented horizontally by asterisks (*) or another character of your choice (what about the first letter of the candidate's name). It may look like this:

```
PERCENT SCALE
BOB     **********
SUE     ******************
BRAD    **********************
MARY    **************
```

3. Show the percents mentioned above by a pie chart with a different color or pattern for each choice. Your output may look like this.



4. Design a board and an interactive game for two people to play TIC TAC TOE. (or other game) Note, the computer is not a player. only the graphic screen is the game board.

5. Create a graphic figure; move the figure around the screen in a set pattern. Example; create a color filled circle; roll the circle around another circle.

6. Trace the pattern made by a bouncing ball dropped from a height H.

7. Mouse in the Maze - Use the file MAZE.DAT
   - Consider the maze as a two-dimensional array MAZE (21,21)
   - Place the mouse in the center by subscripts I=11 and J = 11
   - A space with zero represents a clear path
   - A space that is not zero represents a blocked path
   - Show the mouse's path as he finds his way out of the maze
   - Be sure to mark his path as he moves so he does not retrace his steps
   - The mouse will exit at MAZE (1,1)
   - Refer to notes and discussion

8. Secret Message - see problem defined on the next page.

Secret Message

Given : A data file called "SECRET" with one number per record. Each number represents a code for a character in the secret message. The number 99 acts as a flag to mark the end of the message.

Problem: Use the message below to develop a pattern for numbered codes for the letters of the alphabet (include blanks also). Then, use the codes that you have developed to decode the message in the "SECRET" file and print out the letters in the message.

Think about class discussions in unit IV. Use techniques discussed to develop an efficient program.

T	R	Y		T	O		D	O		T	H	I	S		P	R	O	B	L	E	M	
14	18	4	98	14	24	98	7	24	98	14	15	17	16	98	22	18	24	3	23	9	25	98

I	N		L	E	S	S		T	H	A	N		T	W	E	N	T	Y	
17	26	98	23	9	16	16	98	14	15	1	26	98	14	8	9	26	14	4	98

S	T	A	T	M	E	N	T	S	
16	14	1	14	25	9	26	14	16	99

WEEK 4: DAY 2 Lecture/discussion Continue Unit V,Begin Unit VI

Continue discussion of one-dimensional arrays and tables; review the "CLASS" file example of parallel arrays. That is the file that was created as a result of the class survey on the first day of lab.

TWO-DIMENSIONAL ARRAYS

Discuss the two-dimensional array as
example 1) a class roll or grade book with rows and columns
2) a sheet of graph paper to draw on where a picture is made up of connected points or filled in grid points.

Slide presentation (and color terminal demonstration during lab) on graphics and color graphics.

Graphics

Primitives: Point, Line, Circle - arc of circle (a little curved piece).

Two-tone graphics
- Reverse screen
- Position cursor
- Blink screen (by area)
- Underlines and borders
- Pattern to lines and areas

Color graphics
- Set color to default standard colors
  or user may mix by hue, saturation and brightness
- Set pattern for lines and filled areas
- Draw point, line, arc with the "paint brush" dipped in the
  color just set

ASSIGNMENT: Read pages 131-138
Do worksheet Unit VI-1
Begin Week 4: Day 3 Lab

WEEK 4: DAY 3 Lab Unit VI Two-dimensional arrays

Continue programs/projects; do lab Tale of the Turtle on next page.

WEEK 4: DAY 4 Lecture/discusion Review Unit IV

Review problem solving strategies:
- Top-down design
- Pseudo code
- Flow charts

WEEK 4: DAY 5 Lab Continue lab problems/projects

WEEK 4: DAY 3 Lab Two-dimensional Arrays and Graphics

(Continue problems previously assigned on single arrays)

Objective: Understanding 2 dimensional arrays
Appreciation of programs with computer graphics
Review of Logic

Problem: The tail (or tale) of the turtle

Once upon a time a turtle took a stroll. The turtle accidently got ink all over its tail. As it walked, it made marks with its inky tail. However, when its tail was up, it did not make a mark. The program assigned will show the trail made as it walked.

1. Copy the file "TURTLE" to use input

2. Imagine the screen as a checker board of squares 21 x 21. This will be represented as a 2 - dimensional array B$(21,21). - (21 rows and 21 columns)

3. There are two variables continuously input from the turtle file.

   D shows direction -1 moves up 1 square
   1 moves down 1 square
   -2 moves left 1 square
   2 moves right 1 square
   when D = zero the turtle stops his walk.

   M shows whether to make a mark M = 0 no mark
   M / =0 mark

   (note your mark could be an "*" in the square)

4. To open the file use

```
OPEN "TURTLE" FOR INPUT AS 1
```

5. To INPUT the information one record at a time

```
INPUT # 1, D, M
```

Remember D and M need not be in arrays. They are tested but not needed again. You will use them and test in a loop like last week's problems with the CLASS file.

6. The turtle will begin his walk from the center of the board. Remember the pointers I for rows and J for columns. Set I and J start values for the center of the board.

WEEK 5: DAY 1 Lab

Continue lab problems/projects.

WEEK 5: DAY 2 Lecture/discussion

Review Unit V - Arrays and Tables- by presentation of the following topics:

Sorting: Selection or exchange sorting algorithm. (note bubble sort is in the textbook pages 138-145). Point out in discussion that parallel arrays and pointers are moved also according to the movement of the sorted key. Program file on system as subroutine module.

Searching: Binary search algorithm. Highlight efficiency over table look-up method when values are in order.

Program available on system as subroutine.

WEEK 5: DAY 3 Lab

Continue lab problems/projects

WEEK 5: DAY 4 Lecture/discussion Review/do Unit I History

Summarize history of hardware and software by class discussion of material gathered by students as they filled in blank time periods from Unit I worksheets 3 and 4. Students should have used the library and picked up some of the facts from occasional MAIL messages at the terminal..

WEEK 5: DAY 5 Lab

Complete any lab problems/projects. Prepare for closing activities.

WORKSHEET UNIT VI - 1 TWO-DIMENSIONAL ARRAYS

The term matrix refers to a two-dimensional array.

Have you observed that lines within the body of a loop are indented?

1. The following program shows how to read data into a matrix.

```
10 DIM S(3,2)
20 FOR I = 1 TO 3
30 FOR J = 1 TO 2
40   READ S(I,J)
50   DATA 12,17,3,19,7,21
60 NEXT J
70 NEXT I
80 END
```

   a. Could line 10 be deleted without affecting the program?

   b. What number will be stored in array element S(2,1) when the two loops are completed?

   c. What will be the value assigned the variable name I when the number 7 in the data list is read?

   d. What would occur if lines 60 and 70 were interchanged?

   e. If the statements in lines 20 and 30 were interchanged as well as those in lines 60 and 70, the loops would still be nested, in what array element would the number 7 in the data list now be stored after the execution of two loops?

2. Write a program to print the following matrix:

```
1   0   0   0   0
0   1   0   0   0
0   0   1   0   0
0   0   0   1   0
0   0   0   0   1
```

Statistics and Operations Research

I. LINE PLOTS

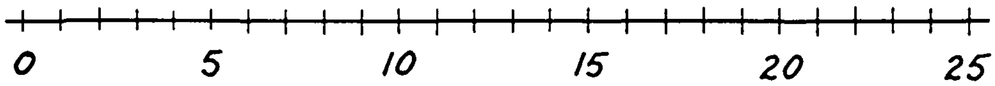
The 1984 Winter Olympics were held in Sarajevo, Yugoslavia. The table below lists the total number of gold, silver, and bronze medals won, by country.

Country	Total Medals	Country	Total Medals
Austria	1	Italy	2
Canada	4	Japan	1
Czechoslovakia	6	Liechtenstein	2
Finland	13	Norway	9
France	3	Sweden	8
Germany, East	24	Switzerland	5
Germany, West	4	USSR	25
Great Britain	1	United States	8
		Yugoslavia	1

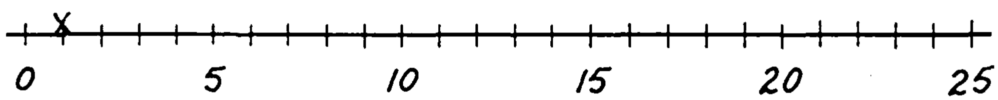
Source: *The World Almanac and Book of Facts,* 1985 edition.

Let's make a *line plot* of these data. First, make a horizontal line.

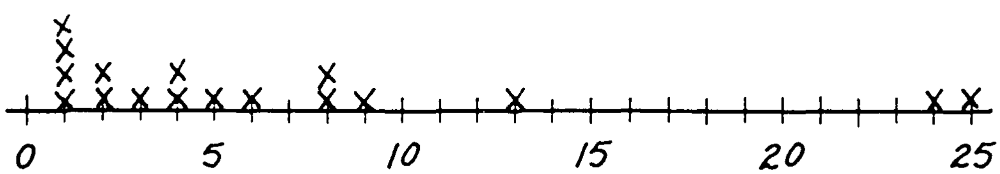
Then, put a scale of numbers on this line using a ruler. Since the smallest number of medals is 1 and the largest is 25, the scale might run from 0 to 25 as shown below.



The first country, Austria, won one medal. To represent Austria, put an X above the line at the number 1.



Continuing this way with the other countries, we can complete the line plot as shown below.



From a line plot, features of the data become apparent that were not as apparent from the list. These features include:

- *Outliers* — data values that are substantially larger or smaller than the other values
- *Clusters* — isolated groups of points
- *Gaps* — large spaces between points

It is also easy to spot the largest and smallest values from a line plot. If you see a cluster, try to decide if its members have anything special in common. For example, in the previous line plot the two largest values form a cluster. They are the USSR and East Germany — both eastern European countries. These two values are quite a bit larger than the rest, so we could also consider these points to be outliers.

Often, we would like to know the location of a particular point of interest. For these data, we might want to know how well the United States did compared to the other countries.

Discussion Questions

1. How many countries won only one medal?
2. How many countries won ten or more medals?
3. Do the countries seem to fall into clusters on the line plot?
4. Describe how the United States compares with the other countries.
5. In this book, you will often be asked to "describe what you learned from looking at the plot." Try to do this now with the plot of medal winners, then read the following sample.

> Seventeen countries won medals in the 1984 Winter Olympics. Two countries, the USSR with 25 and East Germany with 24, won many more medals than the next country, Finland, with 13. The remaining countries were all clustered, with from 1 to 9 medals each. The United States won 8 medals, more than 11 countries but not many in comparison to the leaders. One noticeable feature about these 17 countries is that, with the exception of the United States, Canada, and Japan, they are all in Europe.
>
> The list does not say how many countries did not win any medals. This might be interesting to find out.

Writing descriptions is probably new to you. When you look at the plot, jot down any observations you make and any questions that occur to you. Look specifically for outliers, clusters, and the other features we mentioned. Then organize and write your paragraphs as if you were composing them for your English teacher. The ability to organize, summarize, and communicate numerical information is a necessary skill in many occupations and is similar to your work with science projects and science laboratory reports.

Application 2

Causes of Death

The United States Public Health Service issues tables giving death rates by cause of death. These are broken down by age group, and the table below is for people 15-24 years of age. It gives death rates per 100,000 population for 16 leading causes of death. As an example, a death rate of 1.7 for leukemia means that out of 100,000 people in the United States aged 15-24, we can expect 1.7 of them will die annually from leukemia.

Cause of Death	Death Rate (per 100,000 people aged 15-24 per year)
heart diseases	2.9
leukemia	1.7
cancers of lymph and blood other than leukemia	1.0
other cancers	3.6
strokes	1.0
motor vehicle accidents	44.8
other accidents	16.9
chronic lung diseases	0.3
pneumonia and influenza	0.8
diabetes	0.3
liver diseases	0.3
suicide	12.3
homicide	15.6
kidney diseases	0.3
birth defects	1.4
blood poisoning	0.2

Source: National Center for Health Statistics, Monthly Vital Statistics Report, August 1983.

1. Of 100,000 people aged 15-24, how many would you expect to die annually from pneumonia and influenza?
2. Of 1,000,000 people aged 15-24, how many would you expect to die annually from pneumonia and influenza?
3. Suppose there are 200,000 people, and 3 die from a certain cause. What is the death rate per 100,000 people?
4. Of 250,000 people aged 15-24, about how many would you expect to die annually from motor vehicle accidents?
5. Construct a line plot of these data. To avoid crowding when plotting the *X*'s, round each death rate to the nearest whole number.
6. Which cause of death is an outlier?

Application 1

Rock Albums

The following list of the top 10 record albums in the first five months of 1985 is based on *Billboard* magazine reports.

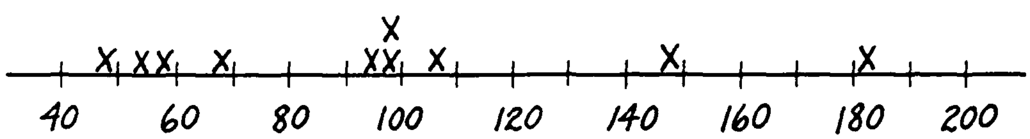
Artist	Title	Total Points
Bruce Springsteen	"Born in the U.S.A."	183
Madonna	"Like a Virgin"	149
Phil Collins	"No Jacket Required"	108
John Fogerty	"Centerfield"	97
Wham!	"Make It Big"	97
Soundtrack	"Beverly Hills Cop"	93
Tina Turner	"Private Dancer"	69
Prince	"Purple Rain"	59
Foreigner	"Agent Provocateur"	54
USA for Africa	"We Are the World"	49

Source: *Los Angeles Times*, May 25, 1985.

The total points were calculated by giving 10 points for each week an album was number 1 on the *Billboard* charts, 9 points for each week it was number 2, 8 points for each week it was number 3, and so forth.

1. If a record was number 1 for 3 weeks, number 2 for 5 weeks, and number 3 for 2 weeks, how many total points would it have?
2. How many points does a record earn by being number 5 for 1 week?
3. If a record was number 4 for 3 weeks and number 5 for 1 week, how many total points would it have?
4. Find two ways for a record to earn 25 points.
5. There were about 21 weeks in the first five months of 1985. Find a way for "Born in the U.S.A." to earn 183 points in these 21 weeks.

The following line plot was constructed from these data.



6. Which record(s) is an outlier?
7. Do the records seem to cluster into more than one group?
8. List the records in the lowest group.
9. List the records in the next lowest group.
10. Write a description of what you learned from studying this plot.

VI. SCATTER PLOTS

The table below gives the box score for the first game of the 1985 National Basketball Association Championship series.

Los Angeles Lakers 114, Boston Celtics 148

LOS ANGELES

	Min	FG-A	FT-A	R	A	P	T
Worthy	37	8-19	4-6	8	5	1	20
Rambis	22	4-6	0-0	9	0	2	8
Jabbar	22	6-11	0-0	3	1	3	12
Magic Johnson	34	8-14	3-4	1	12	2	19
Scott	30	5-14	0-0	2	0	2	10
Cooper	24	1-5	2-2	2	2	3	4
McAdoo	21	6-13	0-0	3	0	5	12
McGee	15	4-7	4-5	2	2	1	14
Spriggs	15	4-7	0-2	3	4	1	8
Kupchak	16	3-3	1-2	2	1	3	7
Lester	4	0-1	0-0	0	1	0	0
Totals	240	49-100	14-21	35	28	23	114

Shooting field goals, 49.0%, free throws, 66.7%

BOSTON

	Min	FG-A	FT-A	R	A	P	T
Bird	31	8-14	2-2	6	9	1	19
McHale	32	10-16	6-9	9	0	1	26
Parish	28	6-11	6-7	8	1	1	18
Dennis Johnson	33	6-14	1-1	3	10	1	13
Ainge	29	9-15	0-0	5	6	1	19
Buckner	16	3-5	0-0	4	6	4	6
Williams	14	3-5	0-0	0	5	2	6
Wedman	23	11-11	0-2	5	2	4	26
Maxwell	16	1-1	1-2	3	1	0	3
Kite	10	3-5	1-2	3	0	1	7
Carr	4	1-3	0-0	1	0	1	3
Clark	4	1-2	0-0	1	3	0	2
Totals	240	62-102	17-25	48	43	17	148

Shooting field goals, 60.8%, free throws, 68.0%

Key for table

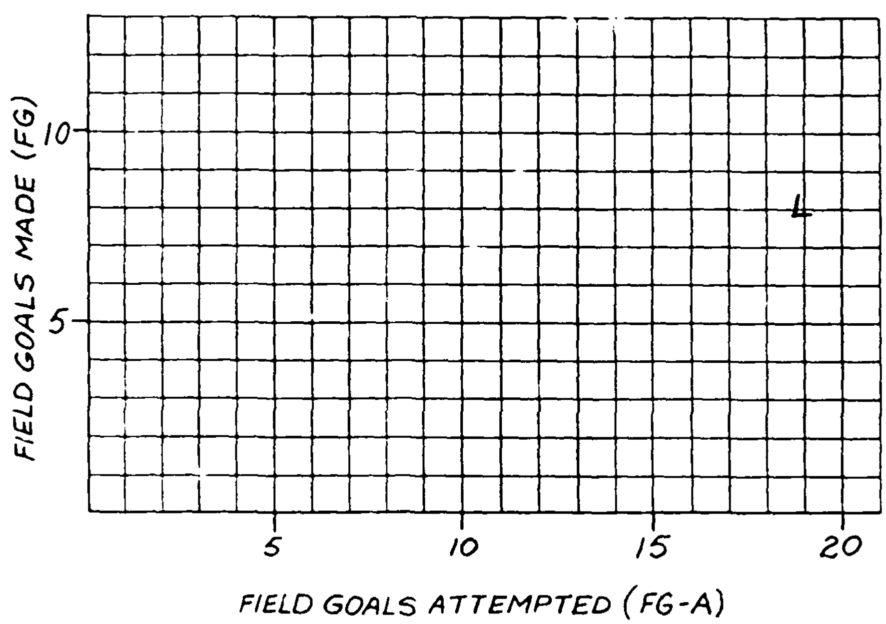
Min	Minutes played
FG-A	Field goals made - field goals attempted
FT-A	Free throws made - free throws attempted
R	Rebounds
A	Assists
P	Personal fouls
T	Total points scored

Source: *Los Angeles Times*, May 28, 1985.

Discussion Questions

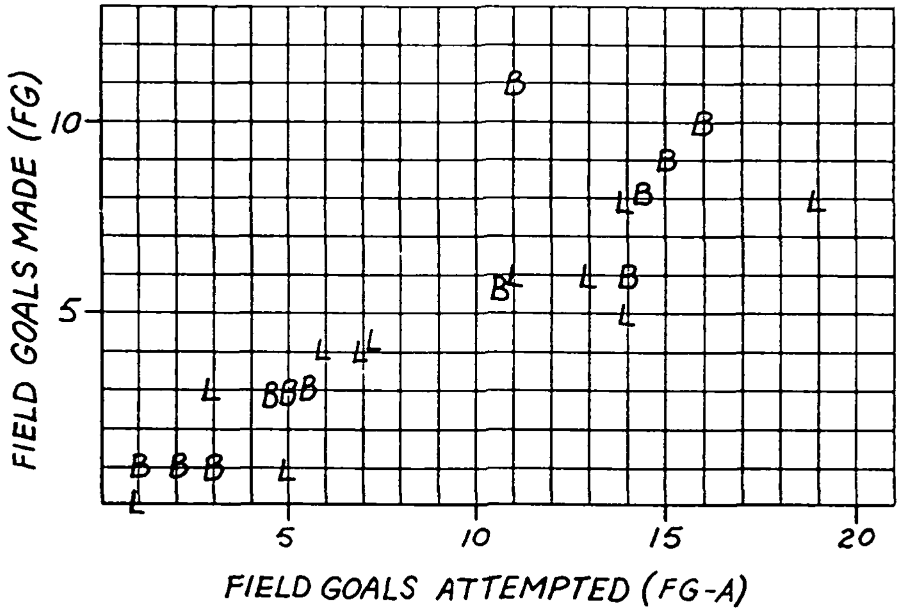
1. How many rebounds did Kevin McHale make?
2. Which player played the most minutes?
3. Which player had the most assists?
4. How many field goals did James Worthy make? How many did he attempt? What percentage did he make?
5. Five players are on the court at one time for each team. Determine how many minutes are in a game.
6. Which team made a larger percentage of free throws?
7. How is the T (total points scored) column computed? Verify that this number is correct for Magic Johnson and for Kevin McHale. (Caution: Some of the field goals for other players were three point shots.)

Do you think that the players who *attempt* the most field goals are generally the players that *make* the most field goals? Of course! We can see this from the box score. To further investigate this question, we will make a *scatter plot* showing field goals made (FG) and field goals attempted (FG-A). First, set up a plot with field goals attempted on the horizontal axis and field goals made on the vertical axis.



Worthy, the first player, attempted 19 field goals and made 8 of them. The L on the preceding plot represents Worthy. The L is above 19 and across from 8. We used an L to show that he is a Los Angeles player.

The completed scatter plot follows. Each B stands for a Boston player and each L for a Los Angeles player.



As we suspected, this plot shows that players who attempt more field goals generally make more field goals, and players who attempt few field goals make few field goals. Thus, there is a *positive* association between field goa: attempted and field goals made.

However, we can see much more from this plot. First, a player who makes every basket will be represented by a point on the line through the points (0, 0), (1, 1), (2, 2), (3, 3), and so forth. Second, the players who are relatively far below this line were not shooting as well as the other players. Finally, we can observe the relative positions of the two teams in this plot.

Discussion Questions

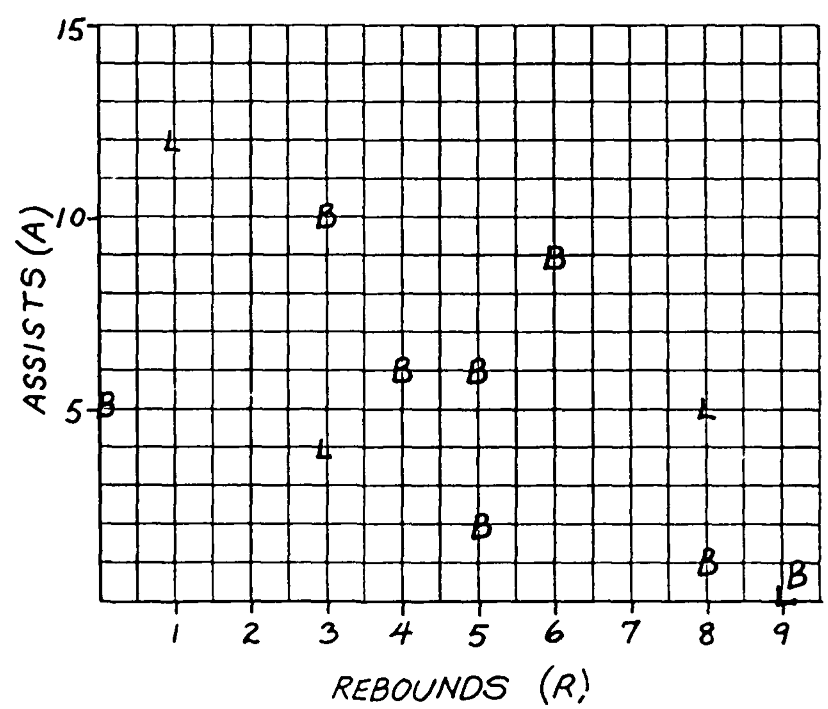
1. Using the scatter plot, find the points that represent the three perfect shooters.
2. Why are all the points below a diagonal line running from lower left to upper right?
3. Is there a different pattern for Los Angeles and Boston players?
4. Which three Laker players were not shooting very well that game?
5. Suppose a player attempts 9 field goals. About how many would you expect him to make?
6. Write a brief description of the information conveyed by this scatter plot. Then read the following sample discussion. Did you notice any information not listed in this sample discussion?

In this plot, we were not surprised to see a positive association between the number of field goals attempted and the number of field goals made. There were three players, two from Boston and one from Los Angeles, who made all the field goals they attempted. One of these Boston players was truly outstanding as he made eleven out of eleven attempts. The Laker players who attempted a great number of field goals generally did not make as many of them as did the Celtics who attempted a great number of field goals. This could have been the deciding factor in the game.

The points seem to cluster into two groups. The cluster on the upper right generally contains players who played over 20 minutes and the one on the lower left contains players who played less than 20 minutes.

An assist is a pass that leads directly to a basket. A player is credited with a rebound when he recovers the ball following a missed shot. Do you think that players who get a lot of rebounds also make a lot of assists? It is difficult to answer this question just by looking at the box score.

To answer this question, we will make a scatter plot showing rebounds (R) and assists (A). This plot includes all players who made at least four rebounds or four assists.

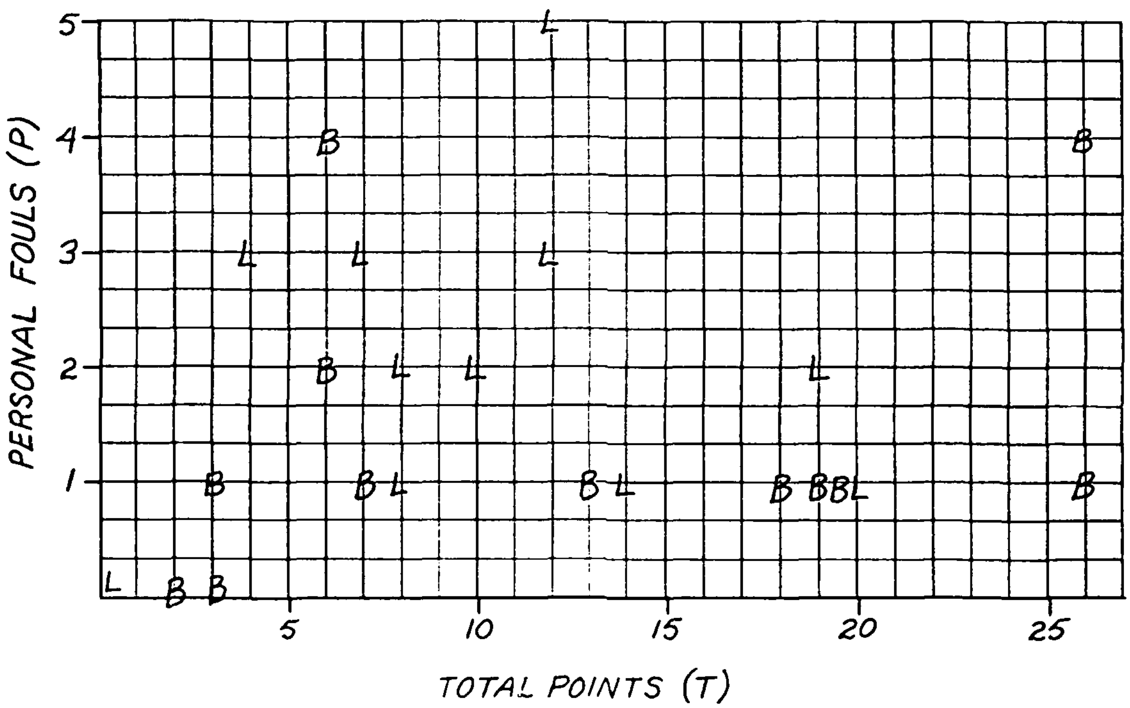


This plot shows that players who get *more* rebounds generally have *fewer* assists, and players who get *fewer* rebounds have *more* assists. Thus, there is a *negative* association between rebounds and assists.

Discussion Questions

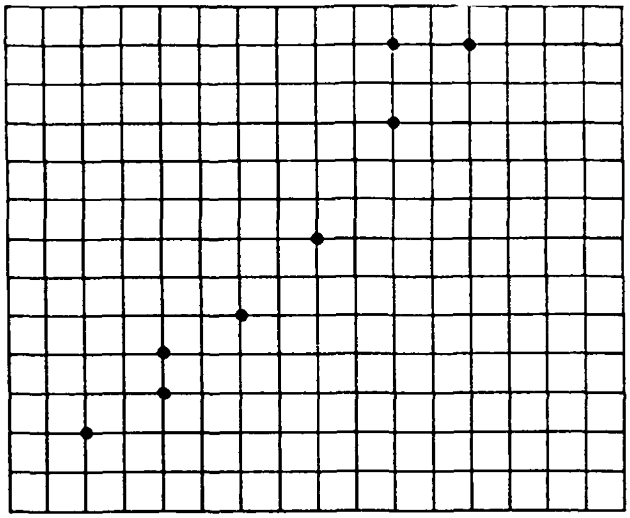
1. Do the players who get the most rebounds also make the most assists?
2. Suppose a player had 7 rebounds. About how many assists would you expect this player to have?
3. Is there a different pattern for Boston players than for Los Angeles players?
4. Why do you suppose players who get a lot of rebounds do not make a lot of assists?
5. If you were the coach and you wanted a player to make more assists, would you instruct him to make fewer rebounds?
6. Why didn't we include players who would have been in the lower left-hand corner of this plot?

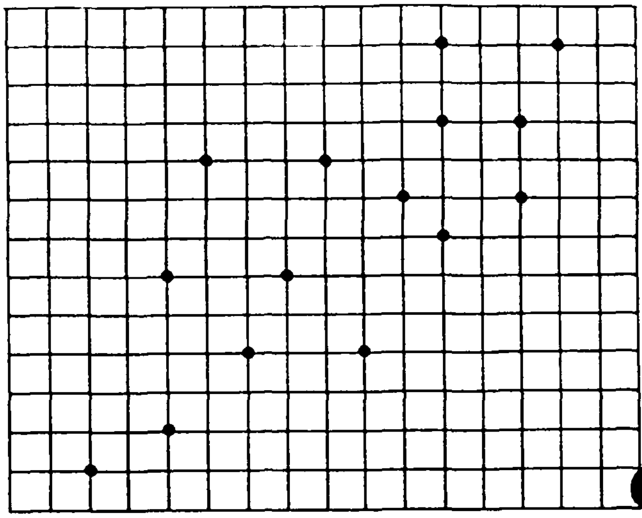
The following scatter plot shows total points and personal fouls for all players.



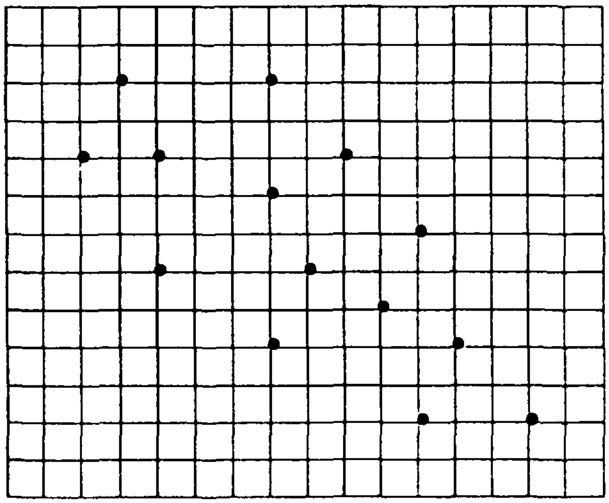
This plot shows *no association* between total points scored and the number of personal fouls committed.

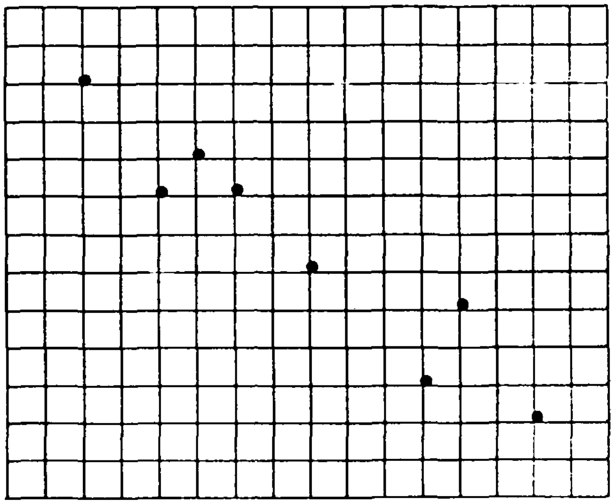
In summary, the following scatter plots show *positive association.*





The following scatter plots show *negative association.*





Application 22

Walk-around Stereos

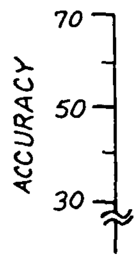
The following table lists 22 "walk-around stereos," each with its price and overall score. The overall score is based on "estimated overall quality as tape players, based on laboratory tests and judgments of features and convenience." A "perfect" walk-around stereo would have a score of 100. Consumers Union says that a difference of 7 points or less in overall score is not very significant.

Ratings of Walk-around Stereos

Brand and Model	Price	Overall Score
AIWA HSP02	$120	73
AIWA HSJ02	180	65
JVC CQ1K	130	64
Sanyo MG100	120	64
Sony Walkman WM7	170	64
Sanyo Sportster MG16D	70	61
Toshiba KTVS1	170	60
JVC CQF2	150	59
Panasonic RQJ20X	150	59
Sharp WF9BR	140	59
Sony Walkman WM4	75	56
General Electric Stereo Escape II 35275A	90	55
KLH Solo S200	170	54
Sanyo Sportster MG36D	100	52
Koss Music Box A2	110	51
Toshiba KTS3	120	47
Panasonic RQJ75	50	46
Sears Cat. No. 21162	60	45
General Electric Great Escape 35273A	70	43
Sony Walkman WMR2	200	41
Sony Walkman WMF2	220	38
Realistic SCP4	70	37

Source: *Consumer Reports Buying Guide*, 1985.

1. Which walk-around stereo do you think is the best buy?
2. A scatter plot will give a better picture of the relative price and overall score of the walk-around stereos. Make a scatter plot with price on the horizontal axis. You can make the vertical axis as follows:



The ≈ lines indicate that part of the vertical axis is not shown, so that the plot is not too tall.

3. Which stereo appears to be the best buy according to the scatter plot?
4. Is there a positive, negative, or no association between price and overall score?
5. Given their overall scores, which walk-around stereos are too expensive?

Application 25

Speeding

The following table shows average freeway speeds as recorded by highway monitoring devices in California. The newspaper gave no explanation why the average speed is missing for 1971 and 1973.

Year	Average Highway Speed in Miles per Hour
1970	59
1971	—
1972	61
1973	—
1974	55
1975	56
1976	57
1977	57
1978	57
1979	58
1980	56
1981	57
1982	57

Source: *Los Angeles Times*, May 22, 1983.

1. Construct a plot over time of the average speeds.
2. Can you guess what year the 55 miles per hour speed limit went into effect?
3. Some people think drivers are ignoring the 55 miles per hour speed limit. Do you think your plot shows that this is the case?
4. The fatalities in California per 100 million miles driven are shown in the following table. Construct a plot over time of these data.

Year	Fatalities per 100 Million Miles
1970	3.8
1971	3.2
1972	3.2
1973	3.0
1974	2.2
1975	2.2
1976	2.3
1977	2.4
1978	2.6
1979	2.5
1980	2.5
1981	2.4
1982	2.1

Source: *Los Angeles Times*, May 22, 1983.

5. Was there a decrease in fatalities when the 55 miles per hour speed limit took effect?
6. Another way to display these data is with a scatter plot of fatalities against speed. Construct such a plot. Place the values for speed on the horizontal axis. Plot the last two digits of the year instead of a dot.
7. What do you learn from the plot in question 6?
8. Why is the plot in question 6 the best one?

VII. LINES ON SCATTER PLOTS

The 45° Line

In the last section we interpreted scatter plots by looking for general relationships of positive, negative, and no association. We also looked for clusters of points that seemed special in some way. This section shows how interpretations of scatter plots are sometimes helped by adding a straight line to the plot. Two different straight lines are used. One is the 45° line going through the points (0, 0), (1, 1), (2, 2), and so forth. The second type is a straight line that is fitted to go through much of the data.

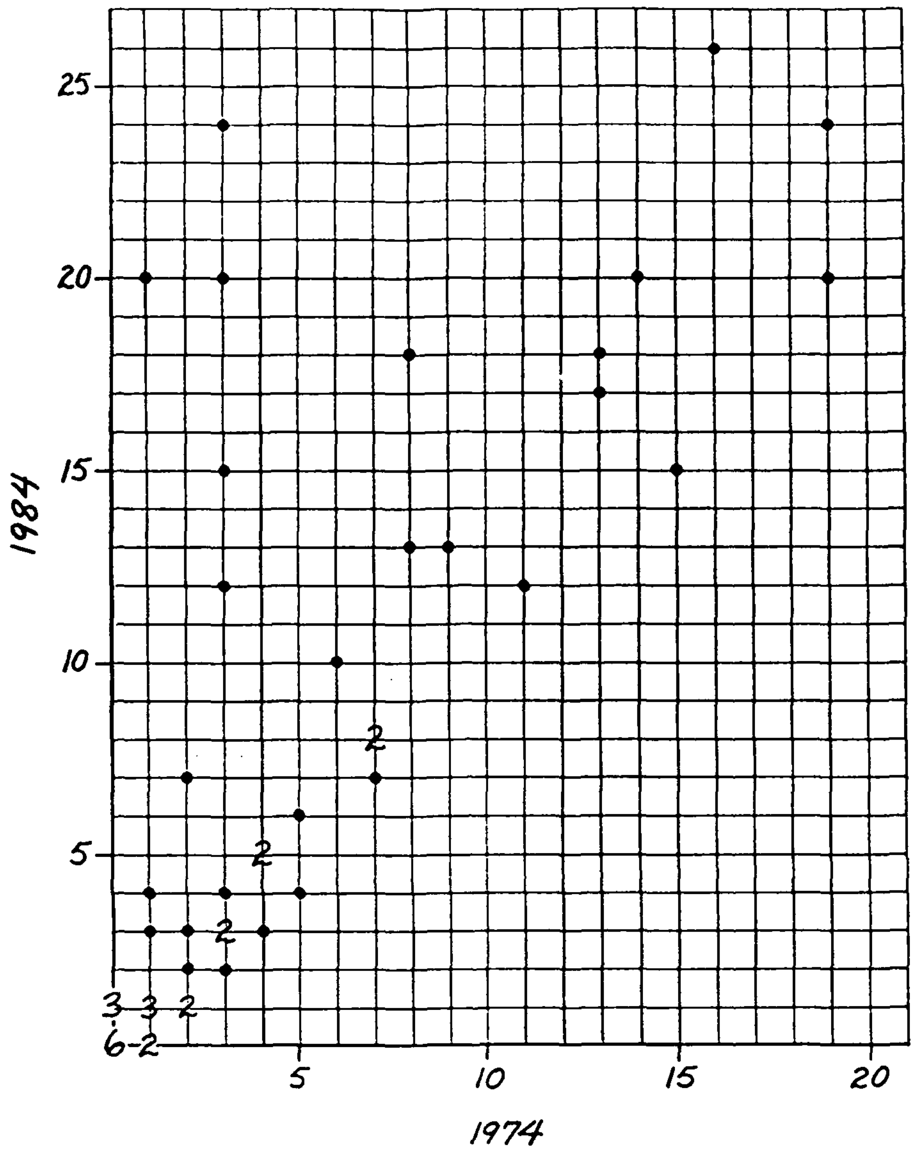
This table lists the number of black state legislators for each state in 1974 and 1984.

Number of Black State Legislators

	1974	1984		1974	1984
Alabama	3	24	Montana	0	0
Alaska	2	1	Nebraska	1	1
Arizona	2	2	Nevada	3	3
Arkansas	4	5	New Hampshire	0	0
California	7	8	New Jersey	7	7
Colorado	4	3	New Mexico	1	0
Connecticut	6	10	New York	14	20
Delaware	3	3	North Carolina	3	15
District of Columbia	n/a	n/a	North Dakota	0	0
Florida	3	12	Ohio	11	12
Georgia	16	26	Oklahoma	4	5
Hawaii	0	0	Oregon	1	3
Idaho	0	0	Pennsylvania	13	18
Illinois	19	20	Rhode Island	1	4
Indiana	7	8	South Carolina	3	20
Iowa	1	1	South Dakota	0	0
Kansas	5	4	Tennessee	9	13
Kentucky	3	2	Texas	8	13
Louisiana	8	18	Utah	0	1
Maine	1	0	Vermont	0	1
Maryland	19	24	Virginia	2	7
Massachusetts	5	6	Washington	2	3
Michigan	13	17	West Virginia	1	1
Minnesota	2	1	Wisconsin	3	4
Mississippi	1	20	Wyoming	0	1
Missouri	15	15	Total	236	382

Source: Joint Center for Political Studies.

The scatter plot of the 1984 number against the 1974 number follows:



A striking feature of the plot is that the points all seem to lie above an (imaginary) diagonal line. Another feature is that there are many points in the lower left-hand corner. In fact, several states sometimes lie at exactly the same point. For example, Arkansas and Oklahoma both lie at (4, 5). To show this, we placed a 2 at (4, 5).

Discussion Questions

1. Place a ruler on the plot next to the line going through (0, 0), (10, 10), (20, 20), and so forth. For states on this line, the 1984 and 1974 numbers of black legislators are equal. How many points are exactly on this line?

2. If a point is above this line, the number of black legislators in that state in 1984 is larger than the number of black legislators that state had in 1974. Name three states for which this statement is true.
3. How many points fall below this line? What can we say about these states? What is the maximum (vertical) distance any of these is below the line? What does this mean in terms of the number of black legislators in 1974 and 1984?
4. Again, consider states above this line, those where the number of black legislators was larger in 1984 than in 1974. What are the names of the 7 or so states that lie farthest above the line? What do these states have in common?
5. The number of black legislators has generally increased from 1974 to 1984. Does this mean that the percentage of legislators who are black has necessarily increased? (Hint: Is the total number of legislators in a state necessarily the same in 1984 as in 1974?)

In summary, this 45° line (sometimes called the $y = x$ line) divides the plot into two regions. We should try to distinguish the characteristics of the points in the two regions. In this plot the top region contains states where the number of black legislators in 1984 is larger than it was in 1974. Most of the states lie in this region. The points in this region that are farthest from the line are those where the number has increased the most from 1974 to 1984. These states turn out to be states in the deep south. There are only a few points slightly below the 45° line, where the number of black legislators was greater in 1974 than in 1984. These are all states that had only 5 or fewer black legislators in 1974. Almost half the states are in the lower left-hand corner, with 5 or fewer in both years. Two states, Illinois and Maryland, had relatively large numbers in both years.

It would have been helpful to plot each state's abbreviation (such as NY for New York) instead of a dot. However, there wasn't room to do this for the states in the lower left corner.

Application 28

Smoking and Heart Disease

The following table lists 21 countries with the cigarette consumption per adult per year and the number of deaths per 100,000 people per year from coronary heart disease (CHD).

Country	Cigarette Consumption per Adult per Year	CHD Mortality per 100,000 (ages 35-64)
United States	3900	257
Canada	3350	212
Australia	3220	238
New Zealand	3220	212
United Kingdom	2790	194
Switzerland	2780	125
Ireland	2770	187
Iceland	2290	111
Finland	2160	233
West Germany	1890	150
Netherlands	1810	125
Greece	1800	41
Austria	1770	182
Belgium	1700	118
Mexico	1680	32
Italy	1510	114
Denmark	1500	145
France	1410	60
Sweden	1270	127
Spain	1200	44
Norway	1090	136

Source: *American Journal of Public Health.*

1. In which country do adults smoke the largest number of cigarettes?
2. Which country has the highest death rate from coronary heart disease?
3. Which country has the lowest death rate from coronary heart disease?
4. If we want to predict CHD mortality from cigarette consumption, which variable should be placed on the horizontal axis of a scatter plot?
5. a) Make a scatter plot of the data.
   b) Draw two vertical lines so there are seven points in each strip.
   c) Place an X in each strip at the median of the cigarette consumption and the median of the CHD mortality.
   d) Do the three X's lie close to a straight line?
   e) Draw in the fitted line.

6. a) Which three countries lie the farthest vertical distance from the line?

   b) How many units do they lie from the line?

   c) Considering the cigarette consumption, are these countries relatively high or low in CHD mortality?

7. If you were told that the adults in a country smoke an average of 2500 cigarettes a year, how many deaths from CHD would you expect?

8. If you were told that the adults in a country smoke an average of 1300 cigarettes a year, how many deaths from CHD would you expect?

9. (For class discussion) Sometimes strong association in a scatter plot is taken to mean that one of the variables *causes* the other one. Do you think that a high CHD death rate could cause cigarette consumption to be high? Could high cigarette consumption cause the CHD death rate to be high? Sometimes, though, there is not a causal relationship between the two variables. Instead, there is a hidden third variable. This variable could cause both of the variables to be large simultaneously. Do you think that this might be the situation for this example? Can you think of such a possible variable?

10. (For students who have studied algebra.) Choose two points on the fitted line, and from them find the equation of the line. Express it in the form $y = mx + b$, where y is mortality from coronary heart disease per 100,000 people (aged 35-64) per year, and x is cigarette consumption per adult per year. Using this equation, how many additional deaths per 100,000 people tend to result from an increase of 200 in cigarette consumption? What number of cigarettes per year is associated with one additional death from CHD per 100,000 people per year?

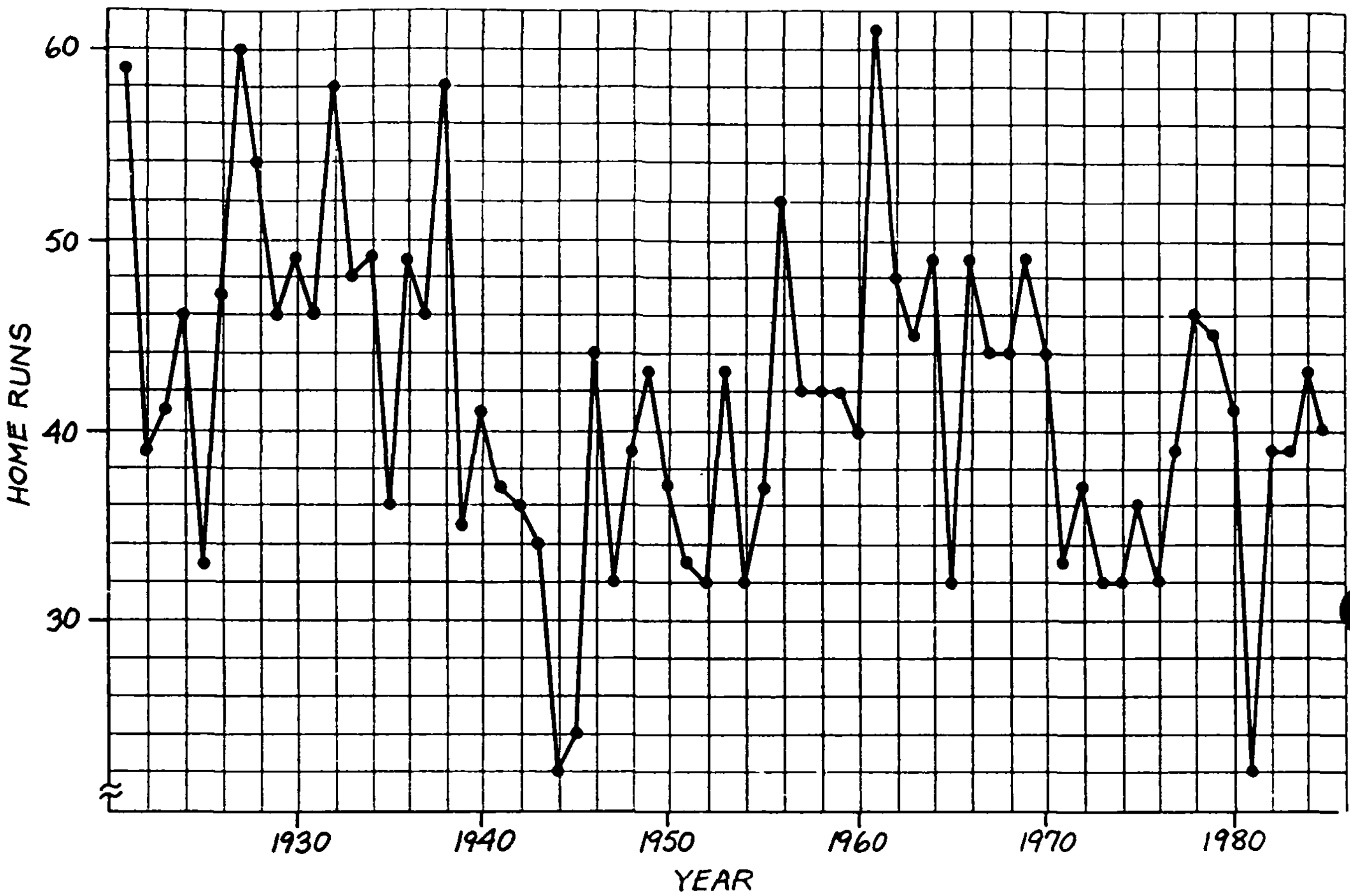
VIII. SMOOTHING PLOTS OVER TIME

The following table lists the American League home run champions from 1921 to 1985.

Year	American League	HR	Year	American League	HR
1921	Babe Ruth, New York	59	1957	Roy Sievers, Washington	42
1922	Ken Williams, St. Louis	39	1958	Mickey Mantle, New York	42
1923	Babe Ruth, New York	41	1959	Rocky Colavito, Cleveland	42
1924	Babe Ruth, New York	46		Harmon Killebrew, Washington	
1925	Bob Meusel, New York	33	1960	Mickey Mantle, New York	40
1926	Babe Ruth, New York	47	1961	Roger Maris, New York	61
1927	Babe Ruth, New York	60	1962	Harmon Killebrew, Minnesota	48
1928	Babe Ruth, New York	54	1963	Harmon Killebrew, Minnesota	45
1929	Babe Ruth, New York	46	1964	Harmon Killebrew, Minnesota	49
1930	Babe Ruth, New York	49	1965	Tony Conigliaro, Boston	32
1931	Babe Ruth, New York	46	1966	Frank Robinson, Baltimore	49
	Lou Gehrig, New York		1967	Carl Yastrzemski, Boston	44
1932	Jimmy Foxx, Philadelphia	58		Harmon Killebrew, Minnesota	
1933	Jimmy Foxx, Philadelphia	48	1968	Frank Howard, Washington	44
1934	Lou Gehrig, New York	49	1969	Harmon Killebrew, Minnesota	49
1935	Jimmy Foxx, Philadelphia	36	1970	Frank Howard, Washington	44
	Hank Greenberg, Detroit		1971	Bill Melton, Chicago	33
1936	Lou Gehrig, New York	49	1972	Dick Allen, Chicago	37
1937	Joe DiMaggio, New York	46	1973	Reggie Jackson, Oakland	32
1938	Hank Greenberg, Detroit	58	1974	Dick Allen, Chicago	32
1939	Jimmy Foxx, Boston	35	1975	George Scott, Milwaukee	36
1940	Hank Greenberg, Detroit	41		Reggie Jackson, Oakland	
1941	Ted Williams, Boston	37	1976	Graig Nettles, New York	32
1942	Ted Williams, Boston	36	1977	Jim Rice, Boston	39
1943	Rudy York, Detroit	34	1978	Jim Rice, Boston	46
1944	Nick Etten, New York	22	1979	Gorman Thomas, Milwaukee	45
1945	Vern Stephens, St. Louis	24	1980	Reggie Jackson, New York	41
1946	Hank Greenberg, Detroit	44		Ben Oglivie, Milwaukee	
1947	Ted Williams, Boston	32	1981	Bobby Grich, California	22
1948	Joe DiMaggio, New York	39		Tony Armas, Oakland	
1949	Ted Williams, Boston	43		Dwight Evans, Boston	
1950	Al Rosen, Cleveland	37		Eddie Murray, Baltimore	
1951	Gus Zernial, Chicago-Philadelphia	33	1982	Gorman Thomas, Milwaukee	39
1952	Larry Doby, Cleveland	32		Reggie Jackson, California	
1953	Al Rosen, Cleveland	43	1983	Jim Rice, Boston	39
1954	Larry Doby, Cleveland	32	1984	Tony Armas, Boston	43
1955	Mickey Mantle, New York	37	1985	Darrell Evans, Detroit	40
1956	Mickey Mantle, New York	52			

Source: *The World Almanac and Book of Facts*, 1985 edition.

From this list it is difficult to see any general trends in the number of home runs through the years. To try to determine the general trends, we will make a scatter plot over time of the number of home runs hit by the champions and connect these points.



This scatter plot looks all jumbled up! It is impossible to see general trends because of the large fluctuations in the number of home runs hit from year to year. For example, 58 home runs were hit in 1938 compared to only 35 the next year. This variation gives the plot a sawtooth effect. The highs and lows, not the overall pattern, capture our attention. To remove the large fluctuations from the data, we will use a method called *smoothing*.

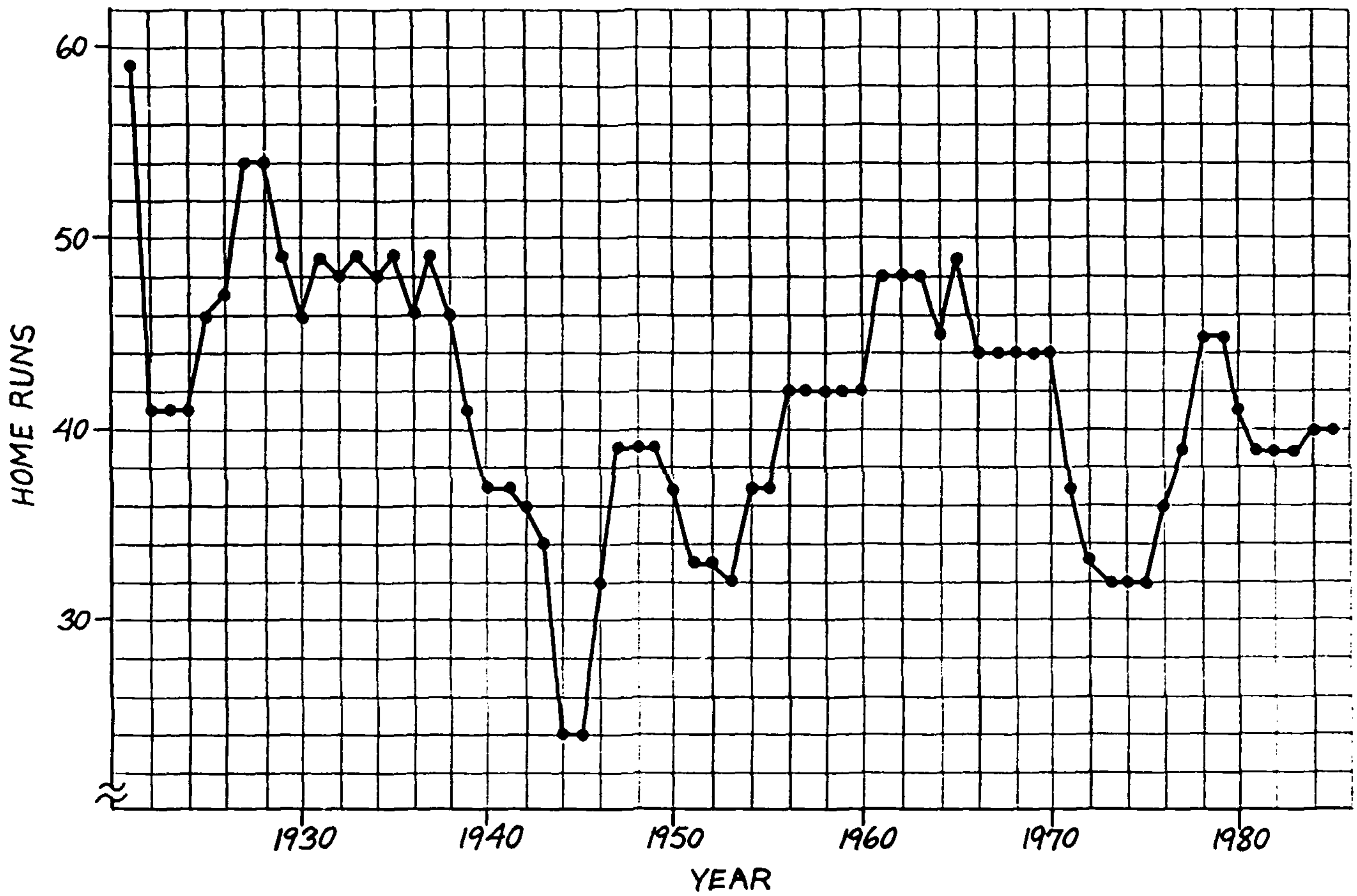
To illustrate, the smoothed version of the first ten years of the home run champions' data follows.

Year	Home Runs	Smoothed Values
1921	59	59
1922	39	41
1923	41	41
1924	46	41
1925	33	46
1926	47	47
1927	60	54
1928	54	54
1929	46	49
1930	49	46
1931	46	

To find the smoothed value for 1924, for example, the 46 home runs for that year are compared to the number of home runs for the year before, 41, and the number of home runs for the following year, 33. The median of the three numbers, 41, is entered into the smoothed values column.

For the first and last years, just copy the original data into the smoothed values column.

The plot of the connected smoothed values follows. Notice what has happened to the large fluctuation between 1938 and 1939. Since this plot is smoother than the previous one, we can see general trends better, such as the drop in the number of home runs in the 1940's.



Discussion Questions

1. Complete the smoothed value column through 1940 for the next ten American League home run champions.

2. Study the smoothed plot of the American League home run champions.

   a. What happened around 1940 that could have affected the number of home runs hit?

   b. Did the increase in the number of games from 154 to 162 in 1961 have an effect on the number of home runs hit?

3. Study the following rule changes. Do any of them seem to have affected the number of home runs hit by the champions?

   1926 — A ball hit over a fence that is less than 250 feet from home plate will not be counted as a home run.

   1931 — A fair ball that bounces over a fence will be counted as a double instead of a home run.

   1959 — New ballparks must have a minimum distance of 325 feet down the foul lines and 400 feet in center field.

   1969 — The strike zone is decreased in size to include only the area from the armpit to the top of the knee.

   1969 — The pitcher's mound is lowered, giving an advantage to the hitter.

   1971 — All batters must wear helmets.

4. In 1981 there was a strike that shortened the season. Can this be seen in the original data? In the smoothed values?

5. Since they were not smoothed, the endpoints may appear to be out of place. The number of home runs hit in 1921 seems too high. Can you determine a better rule for deciding what to write in the smoothed values column for the endpoints?

6. Imagine a curve through the smoothed values. Try to predict the number of home runs hit in 1986.

7. Some students feel that smoothing is not a legitimate method. For example, they do not like changing the original 33 home runs in 1925 to 46 home runs on the plot of smoothed values. Write a description of the trends that are visible in the smoothed plot that are not easily seen in the original plot. Try to convince a reluctant fellow student that smoothing is valuable. Then study the following answer. Did you mention features we omitted?

The original plot of the time series for home runs gives a very jagged appearance. There were values that were quite large for two years in the 1920's, two years in the 1930's, and also in 1961. Extremely low values occurred in the mid-1940's and in 1981. Using this plot, it is difficult to evaluate overall trends. However, the values in the 1940's and early 1950's seem lower than the values in the late 1920's and 1930's.

We get a stronger impression of trends from the smoothed plot of the home run data. In particular, for the years from 1927 to 1935, the values are generally higher than at any other time before or since. The only period that was nearly comparable was in the early 1960's. The original data show that the champions causing the earlier values to be large were Babe Ruth, Jimmy Foxx, and Lou Gehrig. In the 1960's, it was Roger Maris and Harmon Killebrew. These players clearly were outstanding home run hitters!

Application 35

Birth Months

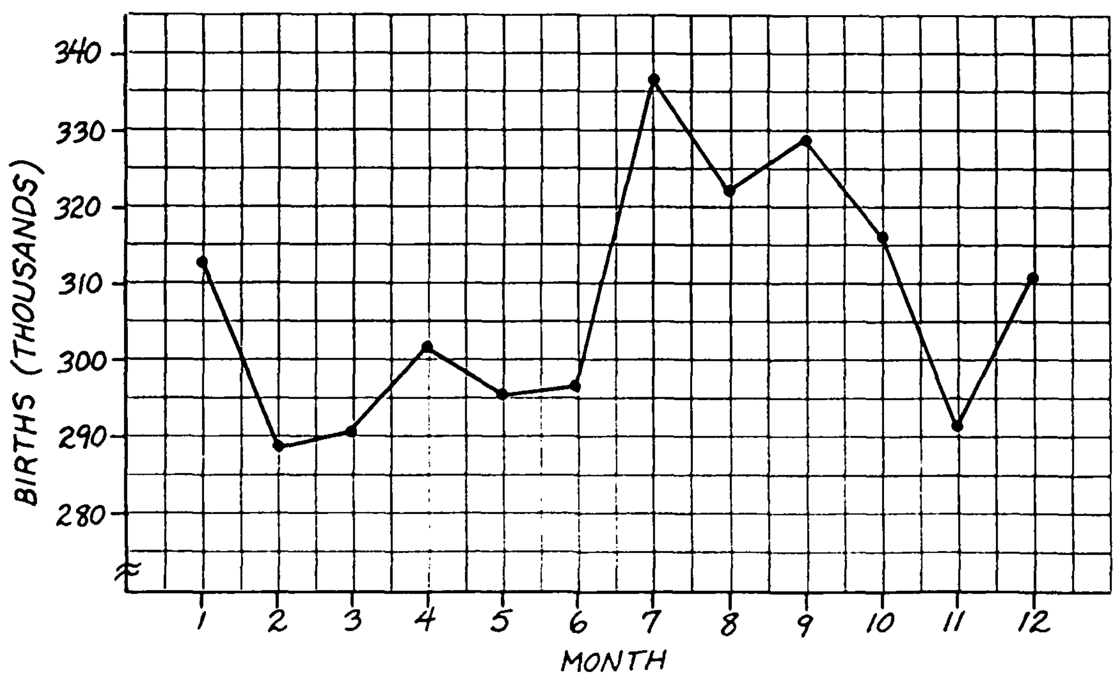
The following table gives the number of babies born in the United States for each month of 1984. The numbers are in thousands.

Month	Births (thousands)	Smoothed Values
January	314	
February	289	
March	291	
April	302	
May	296	
June	297	
July	336	
August	323	
September	329	
October	316	
November	292	
December	311	

Source: National Center for Health Statistics.

1. *How many babies were born in May 1984?*
2. In which month were the most babies born?

The time series plot for these data is given as follows. This plot is a good candidate for smoothing because of the sawtooth effect. This appearance is an indication that some points are unusually large or small.



3. Copy and complete the "Smoothed Values" column.

4. Make a scatter plot of the smoothed values.

5. What is the general trend in the number of babies born throughout the year?

Application 36

Olympic Marathon

The following table shows the winning times for the marathon run (slightly more than 26 miles) in the 1896-1984 Olympics. The times are rounded to the nearest minute.

Year	Winner Name, Country	Time		Time in Minutes	Smoothed Values
1896	Loues, Greece	2 hours	59 minutes	179	
1900	Teato, France	3	0	180	
1904	Hicks, U.S.A.	3	29	209	
1908	Hayes, U.S.A.	2	55	175	
1912	McArthur, South Africa	2	37	157	
1920	Kolehmainen, Finland	2	33	153	
1924	Stenroos, Finland	2	41	161	
1928	El Ouafi, France	2	33	153	
1932	Zabala, Argentina	2	32	152	
1936	Son, Japan	2	29	149	
1948	Cabrera, Argentina	2	35		
1952	Zatopek, Czechoslovakia	2	23		
1956	Mimoun, France	2	25		
1960	Bikila, Ethiopia	2	15		
1964	Bikila, Ethiopia	2	12		
1968	Wolde, Ethiopia	2	20		
1972	Shorter, U.S.A.	2	12		
1976	Cierpinski, East Germany	2	10		
1980	Cierpinski, East Germany	2	11		
1984	Lopes, Portugal	2	9		

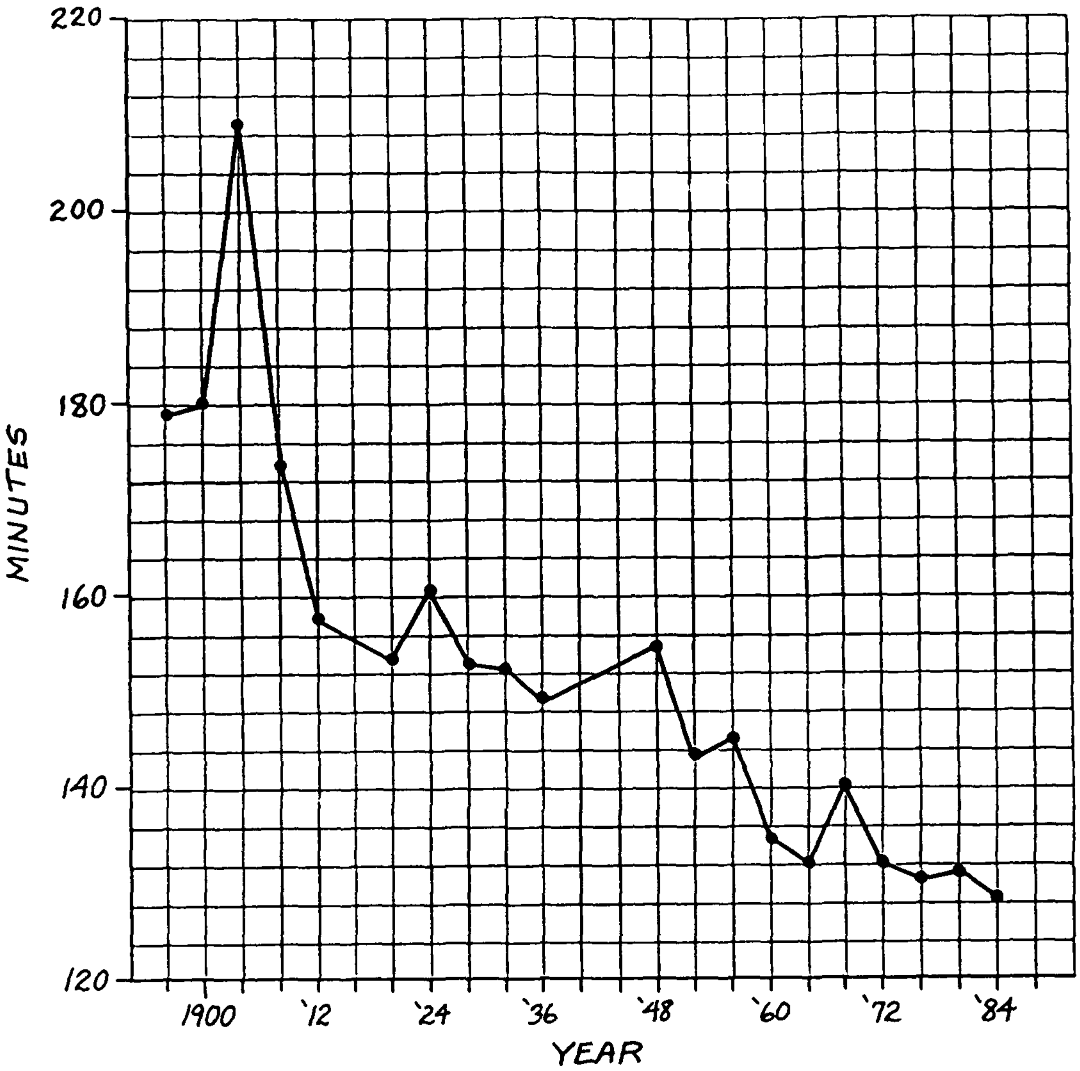
Source: *The World Almanac and Book of Facts,* 1985 edition.

1. The first Olympic women's marathon was not held until 1984. The winner was Joan Benoit of the United States with a time of 2 hours 25 minutes. What was the first year that a Olympic men's marathon winner was able to beat this time?

2. Find the three years when the Olympics were not held. Why were the Olympics not held in these years?

3. Complete the second to the last column of the previous table by converting each time to minutes. The first ten are done for you.

A plot over time with year on the horizontal axis and time in minutes on the vertical axis is shown as follows:



4. What trends do you see in this plot?
5. On the time series plot, which year is farthest from the general trend?
6. Complete the last column of the previous table by smoothing the "time in minutes" column.
7. Construct a plot over time for the smoothed values.
8. Study your plot over time for the smoothed values.
   a. When did the largest drop in time occur?
   b. What do you predict for the winning time in the 1988 Olympic marathon?
   c. Describe the patterns shown on your plot in a short paragraph.

APPENDIX D

A Summer Program in Mathematics and Computer Science
Closing Activities
1:00 - 2:30 P.M.
July 27, 1990

Introduction	**Professor Bernis Barnes** Program Director
Career Awareness Forum	**Professor Gail Finley** Moderator
Participants	**Mrs. Carmella Watkins** Meteorologist National Meteorological Center National Weather Services National Oceanic & Atmospheric Administration
	Mr. Ari Rosner (85) Foreign Service (So.) Amherst College
	Miss Dawn Carroll (85) Architectural Engineer(Jr.) Penn State University
	Mr. Erik Harris (83) Mathematics (Sr.) Naval Academy
	Dr. J. Arthur Jones Mathematician President Futura Technologies, Inc.
Question/Answer Period	
Remarks	**Dr. Marc J. Lipman** Office of Naval Research Department of the U.S. Navy
	Dr. Philip L. Brach Dean College of Physical Science Engineering & Technology University of D.C.
Presentation of Certificates	**Prof. Barnes and Dr. Lipman**

This is to certify that

---

has successfully completed the ONR-UDC
Summer Program in Mathematics and Computer Science
for Academically Oriented Students held at the
University of the District of Columbia,
25 June through 27 July, 1990.

President
University of the District of Columbia

Chief of Naval Research

Project Director,
Summer Program in Mathematics
and Computer Science

Scientific Officer
Math- Science Division
Office of Naval Research

APPENDIX E

Demographic Data

Name____

Home Address____
Street Address

____
City Zip Code

Telephone Number____

School (1989-1990)______________________________________

School (1990-1991)______________________________________

Date of Birth____

Place of Birth____

Are you a U.S. Citizen__________

Race/Ethnicity: (Voluntary) ( )Black ( )White ( )Hispanic ( )Asian ( )Other

Intended Occupation_____________________________________

Parents/Guardians______________________________________

Address (if different from yours)____________________________

____

Telephone Number (if different from yours)______________________

Occupation of Mother_____________________________________

Occupation of Father_____________________________________

University of the
District of Columbia

VAN NESS CAMPUS
4 CONNECTICUT AVENUE, N.W.
WASHINGTON, D.C. 20008

Student Questionnaire

SUMMER PROGRAM IN MATHEMATICS AND COMPUTER SCIENCE

This evaluation is designed to help improve the summer program based on your experience. Please answer each question honestly and according to the directions. Feel free to make any comments to clarify your response in the space below each question.

ENCIRCLE THE RESPONSE OF YOUR CHOICE:

1. Has this program helped to increase your appreciation of mathematics and computer science? YES NO

   Comments (How? or Why not?):

2. Has this program helped to increase your understanding of mathematics and computer science? YES NO

   Comments (How? or Why not?)

3. Has this program helped to increase your awareness of career opportunities and mathematics in the world of work? YES NO

   Comments (How? or Why not?)

4. Will you be able to perform better in mathematics when you return to school as a result of this experience? YES NO

   Comments (Why, How, or Why not?)

5. Has this program experience inspired you to pursue the more challenging math courses in high school? YES NO

   Comments (Which ones? or Why not?)

6. Did you learn to reason more clearly in mathematics this summer? YES NO

Comments (How can you tell?)

7. Was this program everything you expected it to be? YES NO

Comments (Why or Why not?)

ENCIRCLE THE ANSWER THAT BEST DESCRIBES YOUR OPINION:

8. The subject matter in this program was

   too easy   just right   too difficult

9. The size of the classes was

   too small   just right   to large

10. Class periods were

   too short   just right   too long

11. Five weeks was

   too short   just right   too long

RATE THE ITEMS BELOW BY THE FOLLOWING SCALE:

a. poor
b. fair
c. good
d. excellent
e. exceptional

____	Teachers	Comments: ________________
____	Assignments	________________
____	Films	________________
____	Tours	________________
____	Eating Facilities	________________
____	Computer Facilities	________________

13. What did you like most about the program?

14. What did you like least about the program?

15. If you would recommend this program to a friend, what would you tell him/her?

NAME OF THE CAREER YOU ARE PREPARING FOR:

____